A Fast Step Pulse PWM For Cascade Multilevel Inverters

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***Abstract*—The step pulse waveform PWM method is a typical solution in high power cascade inverters. A big problem is to solve transcendent equations, that cause much consuming time. Then an off-line approach is commonly acceptable in practice. In contrary, an on-line approach would be a more elegant solution and no large memmory is needed. This paper presents a simple on-line step pule PWM method, which shows be much superior to present on-line methods for its simplicity, linearity control characteristics and entire modulation range. The mathematical formulations and simulation results have been presented.**

Keywords- cascade multilevel inverter, step pulse PWM, overmodulation

I. INTRODUCTION

An intensive use of high power multilevel inverters in industry applications and power system and mathematical troubles of present PWM algorithms requires the researchers for a further improving of both the computational time duration and related PWM performances. One of the essential problems in PWM methods for that high power applications is to reduce the switching losses of power converters. The step pulse PWM has been well-known and proved be superior for a less number of switchings in a fundamental period. Unfortunately, this technique requires a burden computational time for solving trigonometrical equations. Another problem is to approximately define initial conditions for solving the equations.

Similar to the selective harmonics elimination PWM in two-level inverters, the solutions of step pulse PWM in multilevel inverters cease exist in overmodulation range of high modulation index. Further more, the step pulse PWM in multilevel inverter becomes more complicated and distinguished from two-level SHE PWM by several interruptions of solution existences in undermodulation range. The obtained diagrams of angular set solutions in the whole range of modulation index present several abnormal ranges, while solutions disappear. Besides that, sophisticated curve diagrams complicate the on-line linearizing capability, in which angular diagrams could be replaced with several straight line sections [1]. A nonlinear control characteristics of fundamental voltages deduced from the method would appear. Recently, the resultant algoritms have been presented [2-3]. To avoid the previous abnormal sections, an appropriate algorithm has been introduced with a certain nonlinear control characteristics. The authors have been trying to extend to the possible maximum modulation range by an injection of tripple harmonics into initial voltage waveforms and this has given rise to a little improvement for modulation index range. Even if the off-line methods have eased the determining of initial conditions and gained a reduction of a computation time, these are still mathematically complicated and uncomfortable because of the use of a look-up table. A successful trying of on-line and linear step pulse PWM method has been presented with the use of triogonomic/inverse trigonomic functions [4]. The drawbacks remain for using several dividing operators of variable modulation index and particular drastical reducing of modulation index range.

In this paper, a novel on-line step pulse modulation for cascade multilevel inverters using the principle control between limit trajectories [5],[6] will be presented. The method is advantageous for its simple algorithms, easy and fast computation and linear and continuous control in the whole modulation index range.

II. THE PROPOSED STEP-PULSEPWM METHOD

A. Principle of conventional STEP-PULSE PWM method

In the PWM method, the switching angles $\theta_1, \ldots, \theta_n$ *, for which* $0 \le \theta_1 < \theta_2 < \cdots < \theta_s \le \pi/2$ are chosen in order to make the first harmonic equal to the desired fundamental voltage V_1 and eliminate the specific higher harmonics.

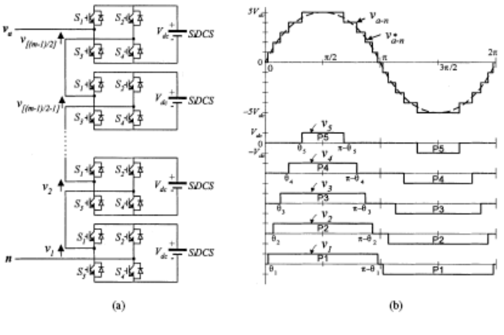

Figure 1. Cascade multilevel inverter- a) one-leg circuit Diagram and b) phase-to-pole voltage

As the application of interest here is a three-phase motor drive, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Consequently, the desire here is to cancel the 5th, 7th, 11th, 13th order harmonics as they dominate the total harmonic distortion.

The mathematical statement of these conditions is then

$$\cos(\theta_1) + \cos(\theta_2) + \cdots + \cos(\theta_s) = V_1/(4V_{dc}/\pi) \quad (1)$$
$$\cos(5\theta_1) + \cos(5\theta_2) + \cdots + \cos(5\theta_s) = 0$$
$$\cos(7\theta_1) + \cos(7\theta_2) + \cdots + \cos(7\theta_s) = 0$$
$$\cos(11\theta_1) + \cos(11\theta_2) + \cdots + \cos(11\theta_s) = 0$$
$$\cos(13\theta_1) + \cos(13\theta_2) + \cdots + \cos(13\theta_s) = 0.$$

This is a system of five transcendental equations in the unknowns $\theta_1, \theta_2, \ldots, \theta_s$ so that at least five steps are needed ($s = 5$) if there is to be any chance of a solution. One approach to solving this set of nonlinear transcendental Eq. (1) is to use an iterative method such as the Newton–Raphson method. The correct solution to the conditions (1) would mean that the output voltage of the 11-level inverter would not contain the 5th, 7th, 11th and 13th order harmonic components. In recent works, step-pulse PWM equations have been converted into polynomial equations and then using resultant theory, complete solutions could be found. Several drawbacks can not be overcome by present step pulse PWM solutions:

Computational algorithms are complicated and for an off-line approach, the look-up tables are needed.

It has been shown that a solution exists for only specific ranges of the modulation index. In case of 11-level cascade inverter with five dc sources, solutions for m exist in a range from 0.376 to 0.846. Among these problems, overmodulation remains unsolvable.

It also can be derived that the solutions are not continuous. For some modulation indices, there are no corresponding solutions. There are more than one set for some modulation indices.

Fig.2a shows all angle solutions and Fig.2b shows the THD factors for these solution sets.

B. The proposed Step Pulse PWM method

The drawbacks of previous methods can be avoided by the proposed SHE-PWM method, which is based on principle control between limit angular trajectories.

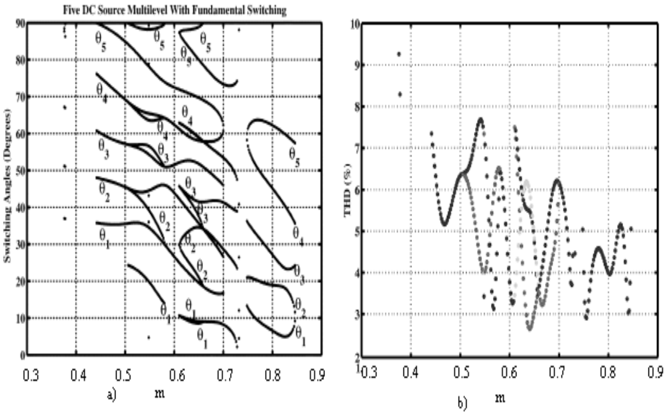

Figure 2. Diagrams the Step pulse PWM obtained from resultant theory of a) different step pulse PWM solutions and b) corresponding THD factors

The step-pulse PWM angle calculation

Let's consider two angle sets ($\theta_{1a}, \theta_{2a}, \ldots, \theta_{sa}$), and ($\theta_{1b}, \theta_{2b}, \ldots, \theta_{sb}$), corresponding to the two limit modulation indices of m_a and m_b. The fundamental voltages and harmonic eliminatiing conditions for both cases can be expressed in the form as follows, x=a,b:

$$\cos(\theta_{1x}) + \cos(\theta_{2x}) + \cdots + \cos(\theta_{sx}) = V_{1x}/(4V_{dc}/\pi) \quad (2)$$
$$\cos(5\theta_{1x}) + \cos(5\theta_{2x}) + \cdots + \cos(5\theta_{sx}) = 0$$
$$\cos(7\theta_{1x}) + \cos(7\theta_{2x}) + \cdots + \cos(7\theta_{sx}) = 0$$
$$\cos(11\theta_{1x}) + \cos(11\theta_{2x}) + \cdots + \cos(11\theta_{sx}) = 0$$
$$\cos(13\theta_{1x}) + \cos(13\theta_{2x}) + \cdots + \cos(13\theta_{sx}) = 0.$$

Let m be the reference modulation index, where $m_a<m<m_b$, and corresponding angle set of ($\theta_1, \theta_2, \ldots, \theta_s$), which satisfy PWM equations (1). The reference angle set, which approximates to exact STEP-PULSE solutions, can be proposed as follows:

$$\cos\theta_j = (1-\eta)\cos\theta_{aj} + \eta\cos\theta_{bj} \quad (3)$$

where

$$\eta = \frac{m - m_a}{m_b - m_a} \quad (4)$$

By setting $\cos\theta_{aj} = a_j$ and $\cos\theta_{bj} = b_j$, where a_j and b_j are constant, the reference angles are approximately calculated by an inverse function,as

$$\theta_j = a\cos\left\lfloor a_j + \eta(b_j - a_j)\right\rfloor \quad (5)$$

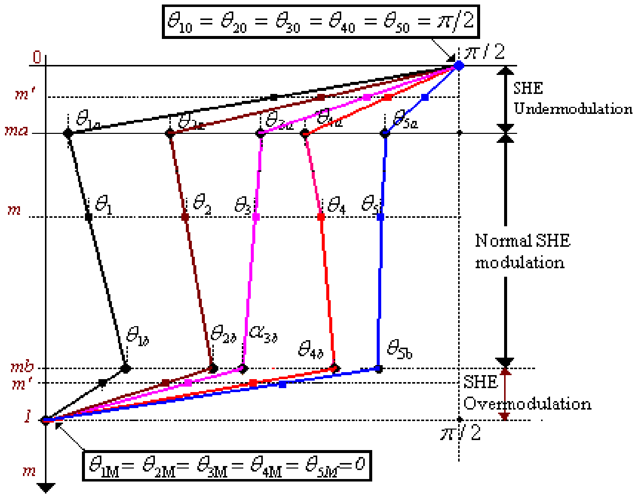

Figure 3. Principle of the proposed STEP-PULSE modulation

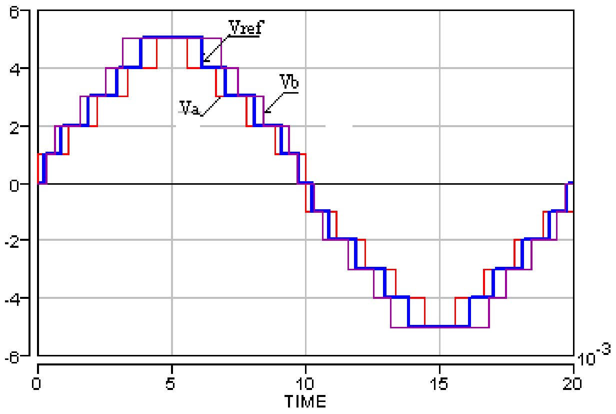

Figure 4. The generating of output phase-to-pole voltage from two limit ones in normal STEP-PULSE modulation

C. *Normal STEP-PULSE modulation:*

In the normal STEP-PULSE modulation range 0.376<m<0.846, there are possible single mode and multi-mode modulations, depending on the number of limit angular sets in use. The method with better THD factor should be with a higher number of limit angular sets.

D. *Single-mode STEP-PULSE overmodulation*

SHE-PWM overmodulation can be also implemented through several modes. Single-mode STEP-PULSE overmodulation could be understood as a direct transition from maximum linear STEP-PULSE PWM trajectory to the trajectory of six step mode.

In fig.4, there is shown angular characteristics in quarter fundamental voltage period. Let's assume that "linear" step pulse PWM is from the limit set of (θ_{1a} ,θ_{2a} ,….,θ_{sa}), to the limit set of (θ_{1b}, θ_{2b},…, θ_{sb}) and step pulse - "overmodulation" is from the limit set (θ_{1b}, θ_{2b},…, θ_{sb}) to the six step mode set as $(\theta_{1M}, \theta_{2M}, \theta_{3M}, \theta_{4M}, \theta_{5M})$.

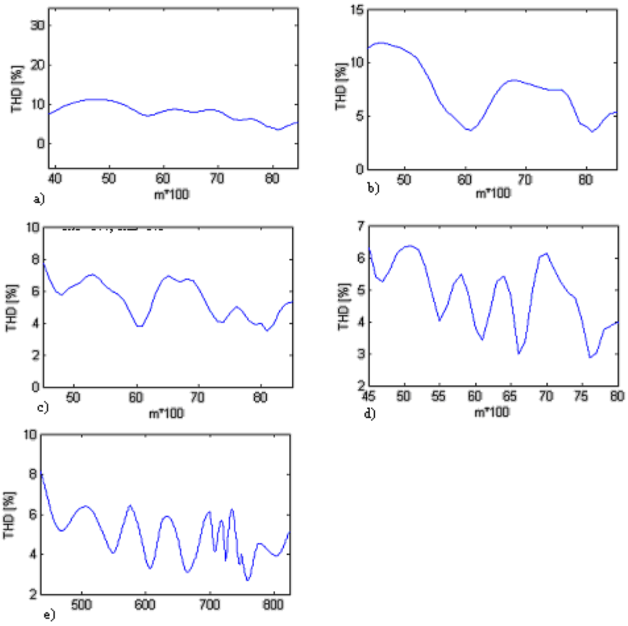

Figure 5. Diagrams of THD factors for various number of limit angular sets a) 2 sets (0.4,0.8), b) 3 sets (0.4,0.6,0.8), c)5 sets (0.4,0.5,0.6,0.7,0.8), d)9 sets (0.4,0.45,0.5,0.55,0.6,0.65,0.7,0.75,0.8) and d) 16 sets (0.4,0.425,0.45,0.475,…,0.8)

At six-step mode, the first angle is set as

$$\theta_{1M} = \theta_{2M} = \theta_{3M} = \theta_{4M} = \theta_{5M} = 0 \qquad (6)$$

The reference angle α_j can be derived by the following formula, notice that $\cos\theta_{jM}=1$:

$$\cos\theta_j = (1-\eta_{bM})\cos\theta_{jb} + \eta_{bM} \qquad (7)$$

where for a six-step mode $m_M = 1$ and

$$\eta_{bM} = \frac{m-m_b}{m_M - m_b} = \frac{m-m_b}{1-m_b} \qquad (8)$$

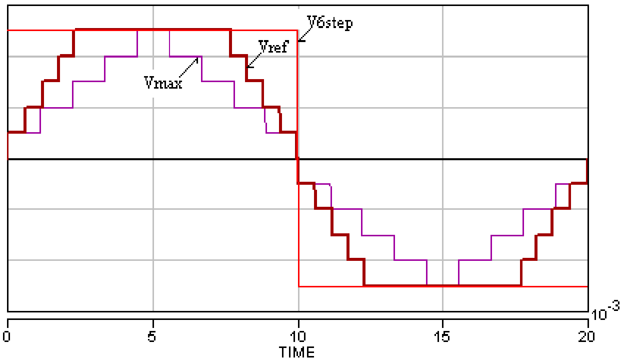

Figure 6. The generating of output phase-to-pole voltage from two limit ones in STEP-PULSE overmodulation

By an inverse function, it can be derived as

$$\theta_j = \cos^{-1}[b_j + (1 - b_j)\eta_{bM}] \tag{9}$$

E. *Single-mode and multi-mode STEP-PULSE undermodulation*

A STEP-PULSE undermodulation can be understood as unsovable STEP-PULSE modulation and this happens for the lower modulation index range. For 11-level inverter, this range can be determined as $0 \leq m \leq 0.376$. Similar to conventional overmodulation, STEP-PULSE undermodulation can be classified as single-mode and multi-mode undermodulation. In a simple case, the proposed single-mode STEP-PULSE undermodulation presents a modulation between two limit angular trajectories, corresponding modulation indices of $m_0=0$ and $m_1=0.376$.

For $m_0=0$, the limit angles can be determined as follows:

$$\theta_{10} = \theta_{20} = \theta_{30} = \theta_{40} = \theta_{50} = \pi/2 \tag{10}$$

Then for reference modulation index m, the reference angles can be derived as follows:

$$\theta_j = \cos^{-1}(\eta b_j) \tag{11}$$

where $\eta = m/0.376$ (12)

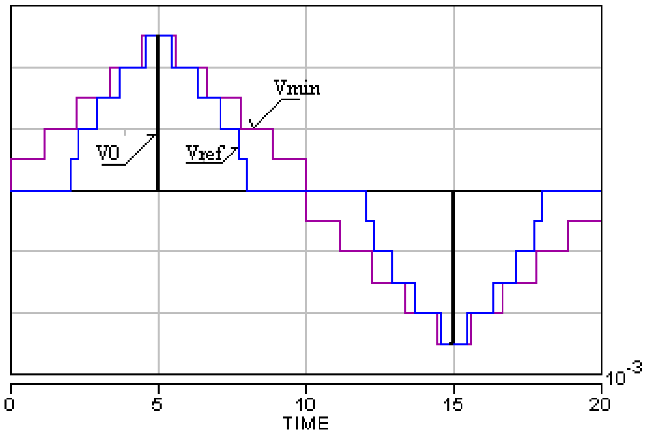

FIGURE 7. The generating of output phase-to-pole voltage from two limit ones in STEP-PULSE undermodulation

In multi-mode undermodulation, the STEP-PULSE PWM transition from modulation index $m_0=0$ to $m_1=0.376$ are implemented in several modes. In Fig.8d, the diagrams of reference angle sets while STEP-PULSE undermodulation are divided into four modes, corresponding to modulation index ranges of (0, 0.1), (0.1,0.2), (0.2,0.3) and (0.3,0.4) .

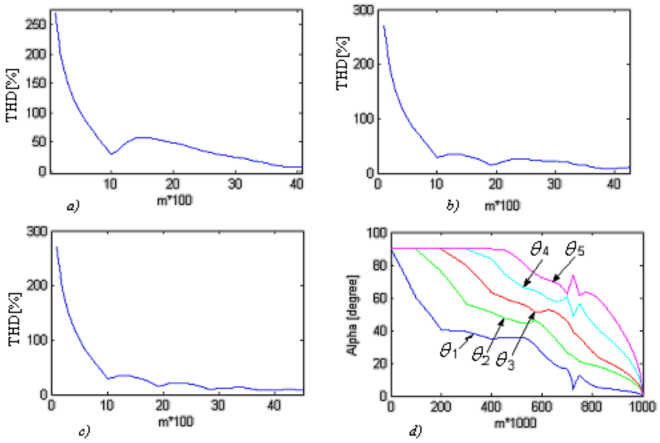

Figure 8. Diagrams of THD factors in STEP-PULSE undermodulation for a) two limit sets (0.1,0.4), b) three limit sets (0.1,0.2,0.4) and c) four limit sets (0.1,0.2,0.3,0.4),and d) diagrams of reference switching angles for entire modulation range with limit sets of (0.1,0.2,0.3,0.4,0.45,0.5,0.55, 0.6,0.65,0.7,0.75,0.8,1)

III. CALCULATING RESULTS

A. *Linear fundamental voltage control*

It can prove easily that (5),(9) and (11) are solutions of (1a) and in these cases, condition of linear control of the output fundamental voltage can be satisfied, that is for a given modulation index m, it can be proved :

$$V_{(1)m} = m(4V_{dc}/\pi) \tag{13}$$

B. *THD factor*

In the normal step pulse PWM: In the investigating process, several variants of angle sets have been considered. With two and three limit sets (m=0.4, m=0.6 and m=0.8), the THD factor can exceed a little a higher value than 10% (Fig.3c and d). With five sets, The THD factor is reduced nearly below 8 %. The THD balance shows good performance with 9 sets , whose THD factor is reduced to acceptable value below 6% (Fig.d). Further increasing of number of angle sets to 12 as in Fig.e does not much reduce the extreme value of THD factor.

In STEP-PULSE undermodulation has been investigated for three cases with the use of two, three and four limit angle sets, corresponding to modulation indices of (0.1,0.4), (0.1,0.2,0.4) and (0.1,0.2,0.3,0.4), respectively. It is shown that the THD factor obtains particular high values for m<0.1. In the range of 0.1<m<0.4, the situation is improved and THD can be reduced to around 35% for m<0.1 and less than 13% for m>0.3.

In STEP-PULSE overmodulation, the STEP-PULSE control algorithm has been implemented with the use of two limit sets of 0.8 and 1. The reference angle sets have been drawn in Fig.8d. The corresponding THD factor has been deduced and drawn in Fig.9. It shows that THD factor can be reduced to about 5% for 0.8<m<0.95. In the remaining range, THD value strongly increases until to six-step mode.

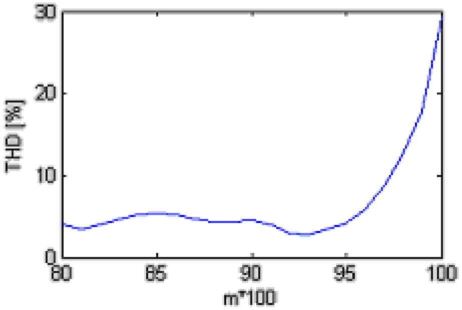

Figure 9. STEP-PULSE overmodulation with two limit sets of 0.8 and 1- Diagram of THD factor

IV. CONCLUSIONS

The paper has presented a novel on-line STEP-PULSE PWM for cascade multilevel inverters. The advantage of the method is simple algorithm, linear control characteristics in the entire range of modulation, including the under- and overmodulation, that presents a big problem in modern modulation techniques. The study shows that with the use of about 6 angular sets, the proposed step-pulse method can obtain a good PWM quality with low THD factor similar to the results from original off-line step pulse PWM.

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