

# AN IMPROVEMENT OF THE DRIVER TRAJECTORY MODEL FOR PATH-PLANNING

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## ABSTRACT

The paper presents some new ideas to improve the accuracy and the generality of the driver trajectory model used in the path-planning module of the NAICC (Navigation Aided Intelligent Cruise Control) system. First, a new Sugeno fuzzy model is developed to calculate the characteristic points, which are dependent on the driver and the road profile, needed to compute the coefficients of a polar polynomial modeling the driver trajectory. The parameters of the membership functions used to represent linguistic variables in Sugeno fuzzy model are estimated by using a Levenberg – Marquardt algorithm so that the mean square error between the observed data and the fuzzy model output is minimized. Second, the effect of the slope and curvature constraints on the shape of the polar polynomial and the driver trajectory are discussed and some nice expressions for estimating these constraints are proposed. Simulation on a wide range of bends with different radii and turn angles shows on the one hand that the obtained fuzzy model is able to calculate the appropriate characteristic points corresponding to the driver profile, and on the other hand that the estimation of the slope and curvature conditions is good enough. An experimental validation with some available test bends shows that the path, planned by using the new trajectory model, fits very well to the real driver trajectory.

## 1. INTRODUCTION

The concept of driver-aid system has been of interest recently because of the need of increasing driver's safety and comfort on the road. Many related projects are currently in progress; one of them is the NAICC (Navigation Aided Intelligent Cruise Control) project. The NAICC system uses a driver's trajectory model to generate reference paths corresponding to different kinds of drivers and to the road profile. In its "Warning Mode", the NAICC system will warn the driver if there is a significant difference between the planned path and the real vehicle trajectory. In its "Vehicle Control Mode", the NAICC system controls the vehicle in order to follow the computed reference path. A basic solution in the determination of the reference path could be to use sequences of line segments and circular-arc segments. However, an attempt to follow such paths results in discontinuities in the steering functions at each junction between the straight line (of zero

curvature) and the circular arc (of constant curvature equal to the inverse of the arc radius). Taking into account the kinetics constraint of practical vehicles, the curvature of the planned path must be continuous because the steering angle cannot be changed suddenly in practice. To ensure the continuous curvature requirement, the polar polynomial is chosen to model the driver trajectory ([1]). The idea is inspired by previous works ([2], [3]), in which polar polynomial is used to generate continuous curvature paths for autonomous car-like robots. In this research, an improvement of the driver's trajectory model discussed in [1] is presented.

The rest of the paper is organized as follows. Section 2 briefly reviews the driver's trajectory modeling principle discussed in the previous work [1]. Section 3 presents the new ideas improving the accuracy and generality of the driver's trajectory model. Section 4 shows the experimental and simulated results validating the new ideas. Section 5 is the conclusion.

## 2. DRIVER'S TRAJECTORY MODELING

In the previous work [1], the authors considered the problem of modeling the trajectory of well-defined classes of drivers, namely the novice drivers (ND) and the very experienced drivers (VED). The behaviours of different kinds of drivers during bend negotiation are not the same; consequently, their driving trajectories are quite different. In this work, we only concentrate on the problem of modeling the ND's trajectory, but it is worth to note that the results can straightforwardly be applied to the class of VEDs.

During bend negotiation, the ND gradually turns the steering wheel, increasing the steering angle to a maximum, and then gradually turns it to the initial position, decreasing the steering angle. This driving technique leads to a progressive and smooth trajectory as shown in figure 1. The feature is that the instantaneous radius of the ND's trajectory in the polar co-ordinate frame has a parabolic shape as illustrated in figure 2. This shape is completely different from that of the VED (see [1] for more details).

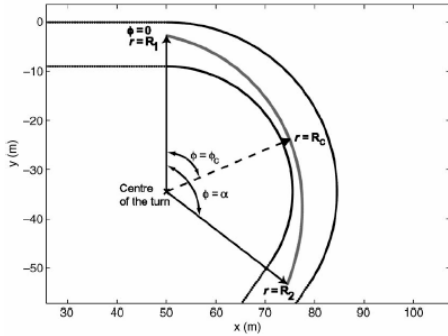


Fig 1: Measured trajectory of the novice driver

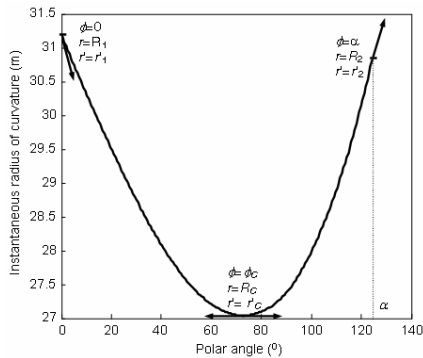


Fig 2: Instantaneous radius of the novice driver's trajectory in the polar co-ordinate frame

Among several types of continuous curvature curves, the polar polynomial is used for modeling the driver's trajectory because it has a closed-form expression. This is particularly convenient in real-time path-planning applications. This idea of trajectory modeling is inspired from the path-planning technique for autonomous guided vehicles in robotics ([2], [3]). The general expression of the polar polynomial is given by:

$$r(\phi) = \sum_{i=0}^n a_i \phi^i \quad (1)$$

The instantaneous curvature of the trajectory generated by (1) is:

$$k(\phi) = \frac{r^2 + 2r'r'' - r.r'''}{(r^2 + r'^2)^{3/2}} \quad (2)$$

The order of the polar polynomial is dependent on the number of constraints that must be satisfied. To generate the specific ND' trajectory described above, the polar polynomial must satisfy the following boundary conditions:

$$\begin{aligned} \text{at } \phi = 0 &\Rightarrow r = R_1, \quad r' = r'_1, \quad k = k_1 \\ \text{at } \phi = \phi_c &\Rightarrow r = R_c, \quad r' = r'_c \\ \text{at } \phi = \alpha &\Rightarrow r = R_2, \quad r' = r'_2, \quad k = k_2 \end{aligned} \quad (3)$$

The position constraints  $R_1, R_c, R_2$  are the radii of the ND's trajectory at the beginning, the vertex and the end of the bend, respectively. These positions are called the characteristic points of the trajectory because those of different kinds of drivers are very different. The slope constraints  $r'_1, r'_c, r'_2$  are the derivatives of the polar polynomial at the mentioned points. The curvature constraints  $k_1, k_2$  are the curvatures of the trajectory at the beginning and the end of the bend. Because there are eight constraints to be satisfied, the polar polynomial modeling the ND's trajectory must be of seventh-order. To calculate the coefficients of the polar polynomial,  $R_1, R_c, R_2, r'_1, r'_c, r'_2, k_1, k_2$  must be computed first. In the paper [1], the constraints are determined as follows.

1. The characteristic points are calculated by using the following equations:

$$R_c = A_0 + \sum_{i=1}^4 A_i R^i + \sum_{i=1}^4 B_i \alpha^i + \sum_{i=1}^2 \sum_{j=1}^2 A_{ij} R^i \alpha^j \quad (4)$$

$$R_1 = R_C + A \quad (5)$$

$$R_2 = R_C + B \quad (6)$$

Where  $R$  is the outer radius of the bend and  $\alpha$  is the turn angle. The parameters of equation (4) are estimated using a least square algorithm. The quantities  $A$  and  $B$  in the equations (5) and (6) are calculated by using a Mamdani fuzzy inference system (FIS), which is manually optimized based on expert knowledge.

There are two recognized limitations of the above expressions. First, the computation of  $R_1$  and  $R_2$  depends on  $R_C$  according to the equations (5) and (6). So if there are any errors in calculating  $R_C$ , the errors will be propagated to  $R_1$  and  $R_2$ . Second, the optimization of the FIS hardly depends on the expert knowledge accumulated. Thus, considering the diversity of existing driving situations, the globalization of the path-planner to a wide range of situations is then difficult. In fact, the FIS has been developed based on real experiments performed on the test track presented in figure 3. Applied to significantly different situations, the FIS generates estimation errors sometimes in the range of a meter, which is not acceptable for a control application.



Figure 3: The test track

2. The slope and curvature constraints play a very important role in determining the driver's trajectory. Over-estimating or under-estimating these values will distort the parabolic shape of the polar polynomial modeling the ND's trajectory. This can lead on the one hand to a planned path which will not be appropriate to represent the driver's behaviour and on the other hand to a trajectory that may go outside the road, and this is of course a dangerous and unexpected situation. In the paper [1], the authors stated that the slope and curvature conditions are dependent on the kinds of drivers and the road profile. Considering the bends in the test track (figure 3), the slope constraints are approximated by linear relationships with the bend radius  $R$  and the curvature conditions are approximated by constants. These simple

expressions cannot be used to determine the slope and curvature constraints in general cases. Because of the mentioned drawbacks, the trajectory model obtained in the previous work can only predict suitable reference paths in some limited road profile configurations. In other words, the model is not general enough. The purpose of this research is thus to obtain a more general and accurate driver's trajectory model by reconsidering the way of calculating the characteristic points, the slope and the curvature conditions. Section 3 presents the new solutions to the mentioned problems.

### 3. IMPROVING THE DRIVER'S TRAJECTORY MODEL

#### 3.1 Identification of the characteristic points

In this work, the characteristic points  $R_1$ ,  $R_2$ ,  $R_C$  are calculated independently from each other by using the following equation:

$$R_x = R_{\min} + A_x \quad (x = 1, 2, C) \quad (7)$$

where  $R_{\min}$  is the inside radius of the bend stored in the database containing the road profile information. This principle has been adopted in order to avoid that the error on the determination of one parameter has an impact on the determination of the other ones as it was the case in the previous solution.

According to equation (7), the identification of the characteristic points  $R_x$  first requires the determination of the quantities  $A_x$  ( $x = 1, 2, C$ ). For this task, it is reasonable to use a FIS to model  $A_x$ . Effectively, by investigating the trajectories recorded from real experiments, we can easily recognize that the quantities are nonlinearly dependent on the radius and the angle of the turn. The membership functions of the FIS are identified from observed data by using a nonlinear least square algorithm.

#### Structure of the Sugeno FIS

Now consider the problem of modeling  $A_x$ . A Sugeno FIS ([4]) is used to describe the relationship between  $A_x$ , the turn radius  $R$  and the turn angle  $\alpha$ . The reason for this choice is that a Sugeno FIS is more flexible and more suitable for numerical training than a Mamdani

one. In addition, the defuzzification methods used with a Sugeno FIS do not need any time-consuming integral calculation and is thus more appropriate to real-time application.

By investigating the recorded trajectories from real experiments, it appears that the nonlinearities between  $A_x$  and the turn angle  $\alpha$  are more important than the one between  $A_x$  and the bend radius  $R$ . As a result, more linguistic values should be defined for the variable  $\alpha$ . Figure 4 presents the membership functions of the fuzzy sets representing the radius  $R$  and the turn angle  $\alpha$ . Note that positions of the membership functions are dependent on some changeable parameters, namely  $b_i$  ( $i=1\dots5$ ). In this work, piece-wise membership functions are used because of their simplicity, but of course, other types of membership functions could be used instead.

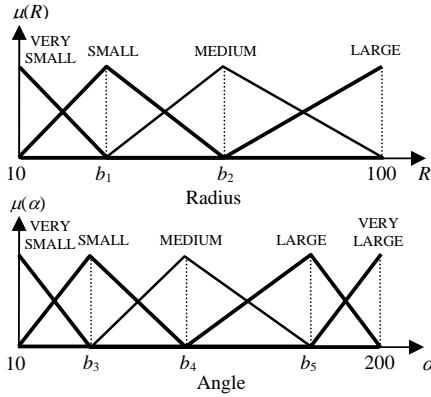


Figure 4: Membership functions of the variables radius and angle

The Sugeno fuzzy rules are of the following general form:

If Radius is  $L_i^R$  and Angle is  $L_i^\alpha$  then  $A_x = a_i$  (8)

where  $L_i^R$  and  $L_i^\alpha$  are linguistic values (e.g. SMALL, MEDIUM, LARGE... as defined in figure 4) of the variables  $R$  and  $\alpha$  respectively, used in the antecedent of the  $i^{\text{th}}$  rule<sup>1</sup>. Using the product operator to implement the fuzzy conjunction and implication, the weighted average method for defuzzification, and finally noting that the fuzzy sets representing the

linguistic values of the variables are fuzzy partitions, the FIS output expression is [5]:

$$A_x(R, \alpha, \mathbf{P}) = \sum_{i=1}^{20} a_i \mu_i(R, \mathbf{b}) \mu_i(\alpha, \mathbf{b}) \quad (9)$$

where  $\mu_i(R, \mathbf{b})$  and  $\mu_i(\alpha, \mathbf{b})$  are the membership functions of fuzzy sets representing the linguistic values  $L_i^R$  and  $L_i^\alpha$  of the bend radius and the turn angle, respectively, in the antecedent of the  $i^{\text{th}}$  fuzzy rule;  $\mathbf{b}=[b_1, \dots, b_5]^T$  is the vector of parameters of the membership functions of the fuzzy sets in the antecedent,  $\mathbf{P}=[a_1, \dots, a_{20}, b_1, \dots, b_5]^T \in \mathfrak{R}^{25}$  is the parameter vector to be estimated from the observed data.

### Estimating the membership functions

Let  $A_x^*(k)$  be the  $k^{\text{th}}$  element of  $N$  observations from real experiments defined by the relation:

$$A_x^*(k) = R_x^*(k) - R_{\min} \quad (10)$$

The error between the observed value  $A_x^*(k)$  and the estimated value  $A_x(k)$  using the FIS is:

$$e(k) = A_x^*(k) - A_x(R(k), \alpha(k), \mathbf{P}) \quad (11)$$

The estimation criterion is defined by:

$$J(R, \alpha, \mathbf{P}) = \frac{1}{N} \sum_{k=1}^N [A_x^*(k) - A_x(R(k), \alpha(k), \mathbf{P})]^2 \quad (12)$$

The estimation step consists in the determination of the parameter vector  $\mathbf{P}$  so that  $J(R, \alpha, \mathbf{P})$  is minimized. Typically, a Newton type iterative algorithm ([6]) is used to solve the above optimization problem:

$$\hat{\mathbf{P}}^{(j+1)} = \hat{\mathbf{P}}^{(j)} - [J''(R, \alpha, \hat{\mathbf{P}}^{(j)})]^{-1} J'(R, \alpha, \hat{\mathbf{P}}^{(j)}) \quad (13)$$

Among the different versions of Newton type algorithms, the Levenberg–Marquardt (LM) algorithm seems to be a reasonable choice because of its fast-convergence and robustness properties. The drawback of an LM algorithm is that it can easily fall into local minimum. One solution to the mentioned problem could be to run the algorithm several times with different parameter initialisation values and to stop when a satisfying result is obtained. Another solution is to use a global search algorithm, e.g. genetic algorithm, but it is beside the scope of this work.

<sup>1</sup>  $i=1\dots20$  because the rule base contains all possible combinations of the linguistic values defined for each variable.

### 3.2 Identification of the slope and curvature constraints

#### Slope constraints

To derive reasonable expressions for estimating the slope constraints, let have a look at the graphical presentation of the slope of the ND's polar polynomial at the characteristic points (figure 5):

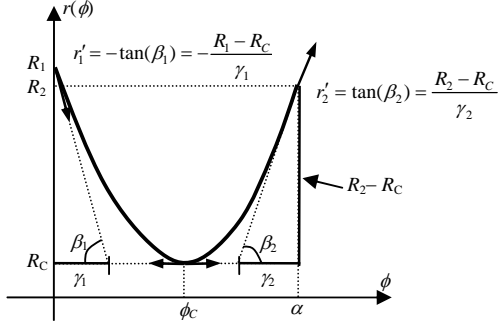


Figure 5: Slope constraints

In the plot, the following relationship can easily be noticed:

$$r'_1 = -\tan(\beta_1) = -(R_1 - R_C)/\gamma_1 \quad (14)$$

$$r'_2 = \tan(\beta_2) = (R_2 - R_C)/\gamma_2 \quad (15)$$

where  $\gamma_1$  and  $\gamma_2$  are directly defined in the figure. The values of  $\gamma_1$  and  $\gamma_2$  are not exactly defined but it is clear that  $\gamma_1$  and  $\gamma_2$  are respectively proportional to the angle  $\phi_c$  and  $(\alpha - \phi_c)$ , where  $\phi_c$  is the angle of the vertex of the turn. Based on this analysis and with the assumption that the characteristic points  $R_1$ ,  $R_2$  and  $R_C$  are correctly estimated, expressions for the slope constraints are proposed as following:

$$r'_1 = -(R_1 - R_C)/(c_1 \phi_c) \quad (16)$$

$$r'_2 = (R_2 - R_C)/[c_2 (\alpha - \phi_c)] \quad (17)$$

The two constants  $c_1$  and  $c_2$  are in a first time arbitrary selected. A suggestion of these constants is  $c_1 = c_2 = 0.01$ .

#### Curvature constraints

In this work, an approximation derived from the expression (2) is used to evaluate the curvature constraints. The exact curvature expression (2) cannot be used directly because it requires the first and second derivative of the polar

polynomial, which are only available once the coefficients of the polar polynomial are determined.

Considering practical situations, to gain comfort, the ND gradually turns the steering wheel during bend negotiation. This behaviour leads to small variations of the first and second derivatives of the polar polynomial. These terms can thus be eliminated from expression (2). As a result, the following approximated expression of the instantaneous curvature is obtained:

$$k \approx 1/r \quad (18)$$

Evaluating the above expression at the beginning and the end of the bend, we have the following curvature constraints:

$$k_1 = 1/R_1 \quad (19)$$

$$k_2 = 1/R_2 \quad (20)$$

**Remark:** The slope and curvature constraints depend on the characteristic points, while the characteristic points are determined based on the driver profile and the road profile by using a FIS. Consequently, the slope and curvature constraints are indeed dependent on the driver and the road profile. So the proposed expressions agree with the basic idea about the slope and curvature constraint stated in [1].

## 4. RESULTS

The proposed improvements discussed above are applied to model the ND's trajectory. First, the Sugeno FIS used for calculating the ND's trajectory characteristic points are identified from experimental data by using a Levenberg - Marquardt optimization algorithm. Data samples used in the estimation and validation steps are collected from real experiments in various bends with different radii and turn angles. Due to the lack of space, the identified Sugeno FIS are not shown in detail here. Table 1 only presents a comparison of the mean square errors obtained on the characteristic points calculated on the one hand with the solution presented in [1], and on the other hand with the one presented in the current work. It is clear that the new algorithm computes the characteristic points more accurately than the earlier one.

Table 1: Mean square error in calculating the characteristic points

	Mean square error (m <sup>2</sup> )		
	$R_1$	$R_2$	$R_C$
New result	0.127	0.184	0.155
Earlier result	3.610	2.852	1.005

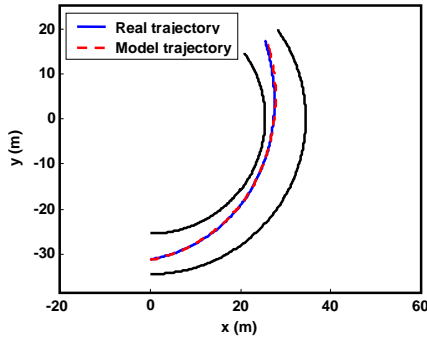


Figure 6: Validation of the driver trajectory with real experiments

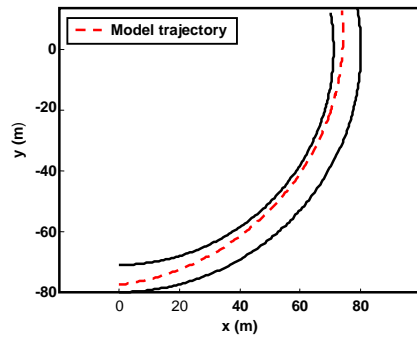


Figure 7: The planned driver trajectory for a simulation bend ( $R = 80\text{m}$ ,  $\alpha = 100^\circ$ )

Once the characteristic points are estimated, they are used to compute the slope and curvature constraints. Then these constraints in combination with the characteristic points are used to determine the coefficients of the polar polynomial (1) modeling the ND's trajectory. The validation phase shows that the obtained driver trajectory model is not only suitable to the bends in real experiments but also to a wide range of simulation bends with different radii and turn angles. In figure 6, the trajectory calculated by using the driver model is almost the same as the one measured during real experiments. Figure 7 shows the planned trajectory for a simulation bend that is significantly different from the ones used in the identification step. From this illustration, it is clear that the planned-path is quite reasonable.

## 5. CONCLUSION

In this work, new algorithms to calculate the characteristic points and the slope and curvature constraints are proposed to improve the accuracy and generality of a driver trajectory model. The characteristic points are calculated independently from each other with the help of Sugeno Fuzzy Inference Systems. These systems are identified from experimental data by using a Levenberg – Marquardt optimization algorithm. The slope and curvature constraints are approximated by some simple explicit expressions in order to be easily computed in real-time path-planning algorithms. In spite of their simplicity, the proposed constrained conditions are well-suited for the determination of the parabolic shape of the instantaneous radius – angle characteristic of the ND's trajectory. The obtained results can be straightforwardly applied to other kind of drivers. Validations on simulated and experimental data show that the trajectory models are accurate and general enough to be used in the path planning module of autonomous driving systems.

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