ANALYSIS OF SERIES IMPEDANCE MATRIX MODELS AND INDUCED MAGNETIC FIELD OF TRANSMISSION LINES ABOVE LOSSY GROUND

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ABSTRACT

This paper considers the influence of earth complex resistivity and sag to magnitude of the induced magnetic field of transmission lines above lossy ground. We first analyze and compare series impedance matrix calculation methods. We secondly propose a simple formula for calculating a height of overhead distribution lines in a span. Finally, we present new theoretical analysis of the induced magnetic field computation of power lines in a span based on the earth-return impedance formulae which proposed by Taku Noda [5]. These analytical formulae are more accuracy about mathematics and physics because, in which, the correction of earth resistivity and the line height are taken into account. Results are presented by means of graphs that are shown that all values of inductances (impedances) and induced magnetic fields always vary with complex resistivity and the height of the line in a span.

Keywords: Transmission lines, induced magnetic field, complex resistivity, line sag.

1. INTRODUCTION

For considering the transient analysis problems of transmission lines, the frequency-dependent ground-return impedance models proposed by Carson [1] is still the standard model for calculating impedance of overhead transmission lines. Based on Carson's formulae, some approximate formulae proposed by Carson-Clem, Sunde [2], Deri *et al* [3], F.Rachidi [4] and Taku Noda [5] are presented. Therefore, based on approximate models this paper will apply and compare various models of ground-return impedances of three-phase distribution lines above lossy ground which has frequency-dependent earth complex resistivity.

The induced magnetic fields generated by overhead transmission lines will effect to materials nearby transmission lines such as communication lines, pipelines and human [6]-[11]. The calculation and measurement of the induced magnetic fields are studied by many authors in the recent years [12]-[17] with different approach and different suppositions of initial data. In calculating, most authors assumed that the earth resistivity is a real number and straight lines parallel to a ground surface. In this paper, we developed on a traditional method for calculation of the induced magnetic field, in which:

- Propose the simple formula for calculating a line height in a span. This purpose is to estimate maximum and minimum values of induced magnetic field of the line in a span.
- Formulate new analytical formulae of induced magnetic field computation of single- and three-phase lines, in which taken into account a complex depth, the frequency-dependent earth resistivity and the function of line height in the span.

Results and comparisons are presented on many graphs, it has been shown that the value of induced magnetic fields are indentical in two cases are taken into account the earth complex resistivitie values measured by Wenner and Driven Rod methods. At midspan of the line the value of induced magnetic field is maximum.

2. SERIES IMPEDANCE MATRIX

In general, at medium frequencies the series impedance matrix of the overhead distribution lines is the sum of three impedances as follows

$$Z_{\Sigma} = Z_{(e)} + Z_{(c)} + Z_{(g)} \qquad (1)$$

Where

- $Z_{(e)}$: the external impedance of the conductor.
- $Z_{(c)}$: the internal impedance of the conductor.
- $Z_{(g)}$: the ground-return impedance of the conductor.

The impedance $Z_{(e)}$ and $Z_{(c)}$ that are the unchanged components and are presented in [15], so in this paper we will only focus on the impedance $Z_{(g)}$.

1. Ground-return self-impedance matrix

Transmission line ground-return impedance matrix has two terms: diagonal term is self impedance and off-diagonal term is mutual impedance. The self impedance Z_{gii} is the ratio of the voltage drop per unit length to the current flowing in the conductor and returning through the earth. The mutual impedance Z_{gij} between *i*-th and *j*-th conductors is the ratio of the induced voltage per unit length in *i*-th conductor to the current in *j*-th conductor. Both the self and mutual impedances are effected by the earth return current is illutrated in Fig. 1.

a. Earth-return self-impedance matrix:

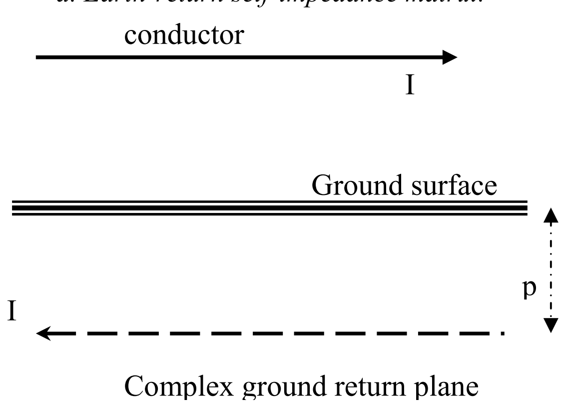

Fig. 1. The ground return path model.

In order to consider the self-impedance matrix of transmission lines, we introduce the calculation models proposed by many authors in following:

The expression of the earth return self impedance of i-th conductor is derived by Carson [1]:

$$Z_{gii,Carson} = \frac{j\omega\mu}{2\pi}.\ln\left(\frac{2h_i}{r_i}\right) + Z'_{gii} \qquad (2)$$

With: $$Z'_{gii} = \frac{j\omega\mu}{\pi}.\int_0^{\infty}\frac{e^{-2.h_i.\xi}}{\xi+\sqrt{\xi^2+j\omega\mu\sigma}}d\xi \qquad (3)$$

The approximate formula is derived by Carson-Clem as follow:

$$Z_{gii,Carson\text{-Clem}} = \pi^2.f.10^{-7} + \frac{j\omega\mu}{2\pi}\ln\left(\frac{D_g}{r_i}\right) \qquad (4)$$

Where

F is the frequency, ρ_g is the earth resistivity and D_g is the fictitious equivalent depth of the earth return path.

$$D_g = 712.\sqrt{\frac{\rho_g}{f}} \qquad (5)$$

The earth return self impedance is proposed by Sunde [2]:

$$Z_{gii,Sunde} = \frac{j\omega\mu}{2\pi}.\ln\left(\frac{2h_i}{r_i}\right) + Z'_{gii} \qquad (6)$$

$$Z'_{gii} = \frac{j\omega\mu}{2\pi}.\ln\left(\frac{1+\gamma_g.h_i}{\gamma_g.h_i}\right) \qquad (7)$$

Where

γ_g: the complex propagation constant in earth.

$$\gamma_g = \sqrt{j\omega\mu.\left(\sigma_g(\omega) + j\omega.\varepsilon_g(\omega)\right)} \qquad (8)$$

The earth return self impedance is proposed by Deri [3]:

$$Z_{gii,Deri} = \frac{j\omega\mu}{2\pi}.\ln\left(\frac{2h_i}{r_i}\right) + Z'_{gii} \qquad (9)$$

and $$Z'_{gii} = \frac{j\omega\mu}{2\pi}.\ln\left(\frac{p+h_i}{h_i}\right) \qquad (10)$$

The earth return self impedance is formulated by Taku Noda [5] as follows

$$Z_{gii,Taku} = \frac{j\omega\mu}{2\pi}.\ln\left(\frac{2h_i}{r_i}\right) + Z'_{gii} \qquad (11)$$

and

$$Z'_{gii} = \frac{j\omega\mu}{2\pi}.\ln\left[\left(1+\frac{\alpha.p}{h_i}\right)^A.\left(1+\frac{\beta.p}{h_i}\right)^{1-A}\right] \qquad (12)$$

In Taku Noda's model, the return current comprises two terms that flowed into two underground surfaces at depths *α.p* and *β.p* as in Fig. 2. The coefficients *A*, *B* and *α, β* are presented in [5].

b. Earth-return mutual-impedance matrix

The expression of the earth return mutual impedance of *i*-th conductor is derived by Carson [1]:

$$Z_{gij,Carson}=\frac{j\omega\mu}{2\pi}.\ln\left(\frac{R_{ij}^{'}}{R_{ij}}\right)+Z_{gij}^{'} \tag{13}$$

$$Z_{gij}^{'}=\frac{j\omega\mu}{\pi}.\int_{0}^{\infty}\frac{e^{-\left(\mathrm{h}_i+h_j\right)\xi}}{\xi+\sqrt{\xi^2+j\omega\mu\sigma}}.\cos\left(d_{ij}.\xi\right)d\xi \tag{14}$$

The earth return mutual impedance is proposed by Deri [3]:

$$Z_{gij,Deri}=\frac{j\omega\mu}{2\pi}.\ln\left(\frac{R_{ij}^{'}}{R_{ij}}\right)+Z_{gij}^{'} \tag{15}$$

$$Z_{gij}^{'}=\frac{j\omega\mu}{4\pi}.\ln\left(\frac{\left(h_i+h_j+2p\right)^2+D_{ij}^2}{\left(h_i+h_j\right)^2+D_{ij}^2}\right) \tag{16}$$

The earth return mutual impedance is proposed by Rachidi [4]:

$$Z_{gij,Rachidi}=\frac{j\omega\mu}{2\pi}.\ln\left(\frac{R_{ij}^{'}}{R_{ij}}\right)+Z_{gij}^{'} \tag{17}$$

$$Z_{gii}^{'}=\frac{j\omega\mu}{4\pi}.\ln\left[\frac{\left(1+\gamma_g.\left(\frac{h_i+h_j}{2}\right)^2+\left(\gamma_g.\frac{D_{ij}}{2}\right)^2\right)}{\left(\gamma_g.\frac{h_i+h_j}{2}\right)^2+\left(\gamma_g.\frac{D_{ij}}{2}\right)^2}\right] \tag{18}$$

The earth return mutual impedance is proposed by Taku Noda [5]:

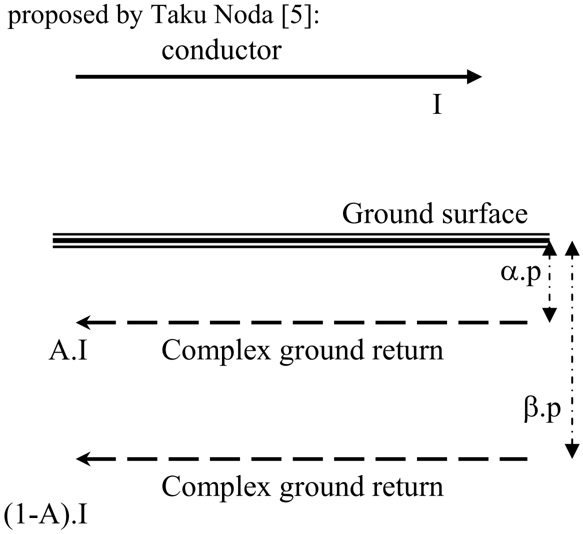

Fig. 2. The ground return path model is presented by Taku Noda

$$Z_{gij,Taku}=\frac{j\omega\mu}{2\pi}.\ln\left(\frac{R_{ij}^{'}}{R_{ij}}\right)+Z_{gij}^{'} \tag{19}$$

$$Z_{gij}^{'}=\frac{j\omega\mu}{4\pi}.\ln\left[\left(\frac{\left(h_i+h_j+2\alpha.p\right)^2+D_{ij}^2}{\left(h_i+h_j\right)^2+D_{ij}^2}\right)^{A}.\left(\frac{\left(h_i+h_j+2\beta.p\right)^2+D_{ij}^2}{\left(h_i+h_j\right)^2+D_{ij}^2}\right)^{1-A}\right] \tag{20}$$

2. *The total series impedance matrix:*

As above presentation the series impedance matrix has three terms which are the external impedance matrix, the internal impedance matrix and the ground return impedance matrix. Five methods for the calculation of the ground-return impedance matrix has been presented.

We will combine these methods to obtain the total series impedance matrix as follows

- Using the external impedance matrix and the internal impedance matrix in [15] combine with Carson's earth return impedance matrix (2) and (13) as

$$Z_{1\Sigma}=Z_{(e)}+Z_{(c)}+Z_{(g)\text{-Carson}} \tag{21}$$

- Using the external impedance matrix and the internal impedance matrix [15] combined with earth return impedance matrix, which has diagonal terms are derived by Carson-Clem's equation (4) and off-diagonal terms are proposed by Rachidi (17) as

$$Z_{2\Sigma}=Z_{(e)}+Z_{(c)}+Z_{(g)\text{-CarsonClem-Rachidi}} \tag{22}$$

- Using the external impedance matrix and the internal impedance matrix [15] combined with earth return impedance matrix, which has diagonal terms are derived by Sunde's equation (6) and off-diagonal terms are proposed by Rachidi (17) as

$$Z_{3\Sigma}=Z_{(e)}+Z_{(c)}+Z_{(g)\text{-Sunde-Rachidi}} \tag{23}$$

- Using the external impedance matrix and the internal impedance matrix [15] combined with Deri's earth return impedance equations (9) and (15).

$$Z_{4\Sigma}=Z_{(e)}+Z_{(c)}+Z_{(g)\text{-Deri}} \tag{24}$$

- Using the external impedance matrix and the internal impedance matrix [15] combined with Taku Noda's earth return impedance equations (11) and (19).

$$Z_{5\Sigma}=Z_{(e)}+Z_{(c)}+Z_{(g)\text{-Taku}} \tag{25}$$

Because components of the external impedance matrix and the internal impedance matrix in total series impedance matrices (21)-(25) are the same, so this paper only compares between earth return impedance matrices that are derived by Carson-Clem, Rachidi, Deri and Taku Noda.

The results, presented in graphs, are examined to 500kV transmission line [18] and the earth resistivity is the frequency dependent complex function [19].

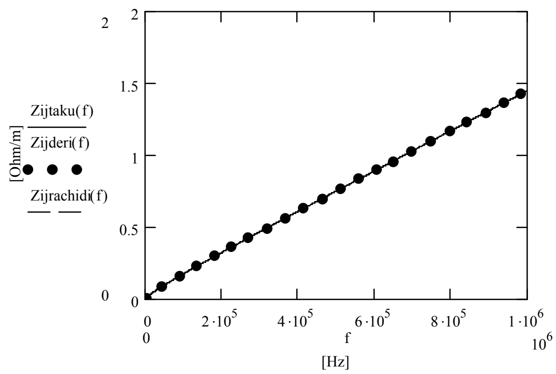

Figure. 3. Comparison between expressions (15), (17) and (19) in frequency domain.

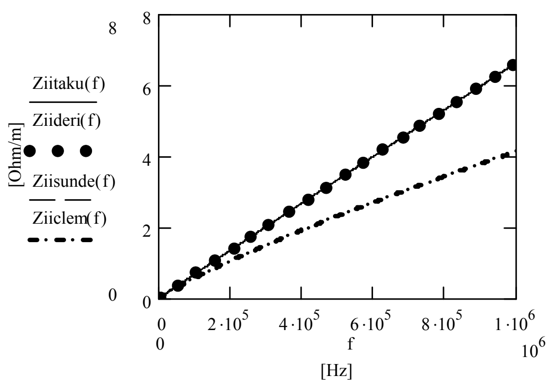

Fig. 4. Comparison between expressions (4), (6), (9) and (11) in frequency domain.

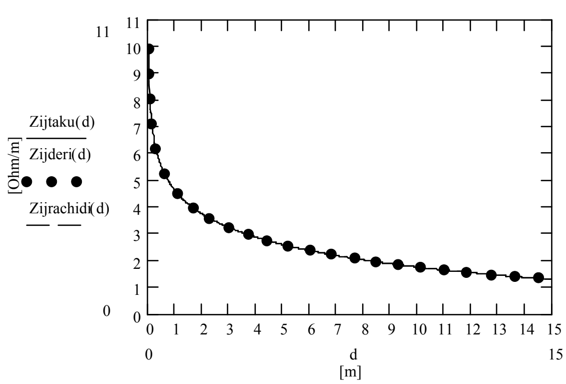

Fig. 5. Comparison between expressions (15), (17) and (19) from horizontal distance D_{ij} between *i*-th and *j*-th phases.

By considering various models of impedance matrices are expressed and illustrated on Figs. 3. – 5. It can be seen that the models are identical. However, Taku Noda's model is better than other ones about mathematical and physical bases because Taku Noda approximated the integral functions (3) and (14) by double-exponential approximations of the coefficients *A*, *B*, *α* and *β* are discussed in [5].

3. A SIMPLE FORMULA OF THE LINE HEIGHT IN THE SPAN

Most authors of previous studies calculated electromagnetic fields of power transmission lines with assume horizontal straight lines parallel to a ground surface with a height is called average height between maximum and minimum height of the line. However, the calculation result of the electromagnetic fields will be different as large as 40% in [20].

Therefore, the height of the line is given by [20]:

$$h(y) = h_{\min} + 2\delta.\sinh^2\left(\frac{y}{2\delta}\right) \tag{26}$$

Where δ is a solution of the equation

$$\frac{S}{2\delta} = \sinh^2\left(\frac{L}{4\delta}\right) \tag{27}$$

H_{max} is maximum height of the line, h_{min} is minimum height of the line, *S* is the sag of the line and *L* is length of the line in a span.

We propose a simple formula to calculate the height of the line in a span [16]:

$$h(y) = h_{\min} + \frac{4.S}{L^2}.y^2 \tag{28}$$

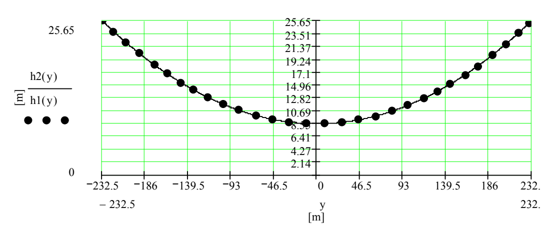

Fig. 6: the 500kV transmission line model in a span [20].

Table .1. is the comparison between (26) and (28), it is shown that maximum error between two formulae is about 1-2%. However, in high voltage engineering, this error is small and we can accept the simple formula (28) for calculating the induced magnetic field of lines in the span.

Table. 1. The comparison between (26) and (28).

y(m)	h(y)↔(26)	h(y)↔(28)
0	8.5	8.5
46.5	9.185	9.186
93	11.24	11.244
139.5	14.667	14.674
186	19.469	19.476
232.5	25.65	25.65

4. NEW ANALYTICAL FORMULAE OF INDUCED MAGNETIC FIELD

By considering and comparing various different models of impedance matrices are presented in Section II, we have seen that Taku Noda's proposed formulae have gained more accuracy using two complex earth-return planes. Therefore, we now chose this model for formulating and calculating the induced magnetic fields of power transmission lines.

New analytical formulae of induced magnetic field components of single- and three-phase lines are given by

a. Induced magnetic field of single-phase transmission lines:

Vertical induced magnetic field component of single-phase line is written as

$$\overline{\mathrm{H}}_x = \frac{I}{2\pi}\left(\begin{array}{l}\dfrac{h_i(y)\text{-}z}{\left(h_i(y)\text{-z}\right)^2+\mathrm{x}^2}+A.\dfrac{h_i(y)+z+2\alpha p}{\left(h_i(y)+\mathrm{z}+2\alpha.\mathrm{p}\right)^2+\mathrm{x}^2}\\ +(1\text{-A}).\dfrac{h_i(y)+z+2\beta p}{\left(h_i(y)+\mathrm{z}+2\beta.\mathrm{p}\right)^2+\mathrm{x}^2}\end{array}\right) \tag{29}$$

Horizontal induced magnetic field component of single-phase line is given by

$$\overline{\mathrm{H}}_z = \frac{I.x}{2\pi}\left(\begin{array}{l}\dfrac{1}{(h_i(y)\text{-}z)^2+x^2}\text{-}\dfrac{A}{(h_i(y)+z+2\alpha.p)^2+x^2}\\ \text{-}\dfrac{1-A}{(h_i(y)+z+2\beta.p)^2+x^2}\end{array}\right) \tag{30}$$

b. Induced magnetic field of three-phase transmission lines:

Vertical induced magnetic field component of three-phase line is expressed as

$$\overline{\mathrm{H}}_{x3\phi} = \frac{\sum I_k}{2\pi}\left(\begin{array}{l}\dfrac{h_k(y)\text{-}z}{\left(h_k(y)\text{-z}\right)^2+\mathrm{x}_\mathrm{k}^2}\\ +A.\dfrac{h_k(y)+z+2\alpha p}{\left(h_k(y)+\mathrm{z}+2\alpha.\mathrm{p}\right)^2+\mathrm{x}_\mathrm{k}^2}\\ +(1\text{-A}).\dfrac{h_k(y)+z+2\beta p}{\left(h_k(y)+\mathrm{z}+2\beta.\mathrm{p}\right)^2+\mathrm{x}_\mathrm{k}^2}\end{array}\right) \tag{31}$$

Horizontal induced magnetic field component of three-phase line is found as

$$\overline{\mathrm{H}}_{z3\phi} = \frac{\sum x_k.I_k}{2\pi}\left(\begin{array}{l}\dfrac{1}{(h_k(y)\text{-}z)^2+x_k^2}\text{-}\dfrac{A}{(h_k(y)+z+2\alpha.p)^2+x_k^2}\\ \text{-}\dfrac{1-A}{(h_k(y)+z+2\beta.p)^2+x_k^2}\end{array}\right) \tag{32}$$

Where:

k : a,b,c – phase a, phase b, phase c of three-phase line.

I_k : k-th phase current [A].

h_k : height of k-th phase conductor [m].

x_k: horizontal distance from k-th phase conductor to observed point [m].

The equations (28)-(31) formulated by authors are more accuracy, in which the complex depth p of constant earth resistyvity is corrected by using the complex resistivity [19] and the constant height of the line is also corrected by the equation (26) or (28).

5. EXAMPLE AND RESULTS

Considering an example of high voltage three-phase distribution line as in [17] above two-layer earth, where earth resistivities are measured by Wenner and Driven Rod methods [19]. Upper layer reistivity *ρ1(DC)* is 100 Ωm and lower layer resistivity *ρ2(DC)* is 1000 Ωm. Rated current I = 610A. . Line height is 10m. Phase-phase distance is 4m.

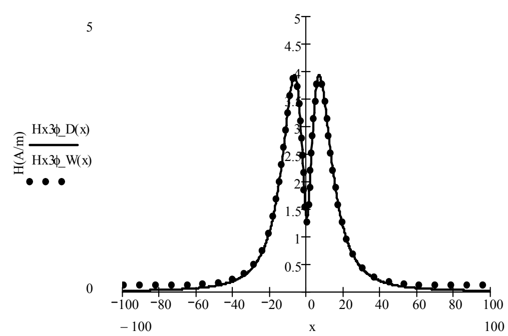

Fig. 7: Horizontal induced magnetic field of 110kV three-phase line at ground (z=0).

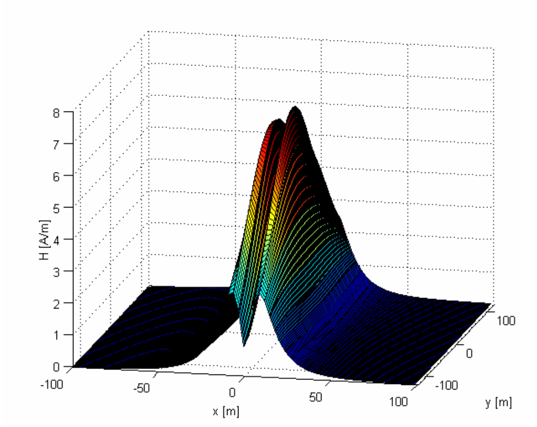

Fig. 8: Horizontal induced magnetic field of 110kV three-phase line at ground (z=0).

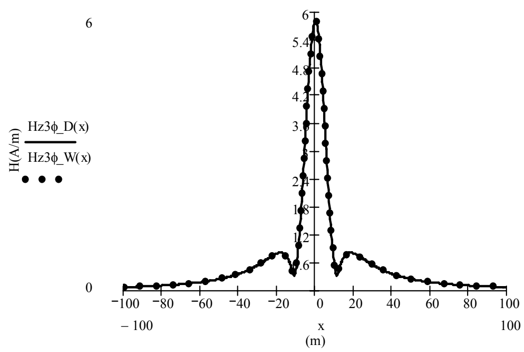

Fig. 9: Vertical induced magnetic field of 110kV three-phase line at ground (z=0).

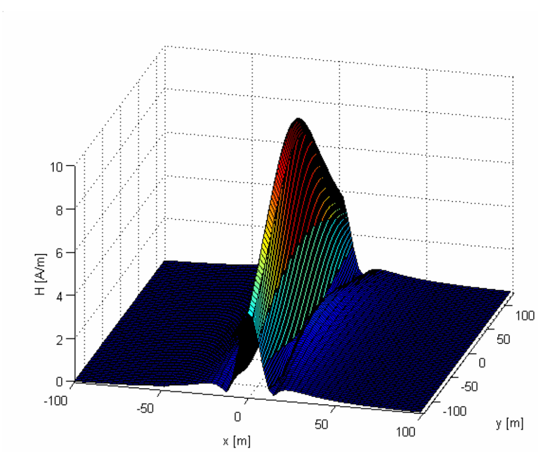

Fig. 10: Vertical induced magnetic field of 110kV three-phase line at ground (z=0).

Figs. 7.- 8. are horizontal induced magnetic field values of the line above lossy ground are calculated in 2D and 3D domains. It is shown that these calculation values are similar with any earth resistivity value that can be measured Wenner or Driven Rod methods. This thing can be seen on Figs. 9. -10. of vertical induced magnetic filed and on Figs. 11. – 12. of total induced magnetic field. The induced magnetic field values of transmission line are largest at midspan and smallest at two poles of line in span. This difference can be larger than 40% because it depends on the length of span, sag in span, tower structure and voltage level... The area of induced magnetic field values is largest at under or near transmission line (around ±20m).

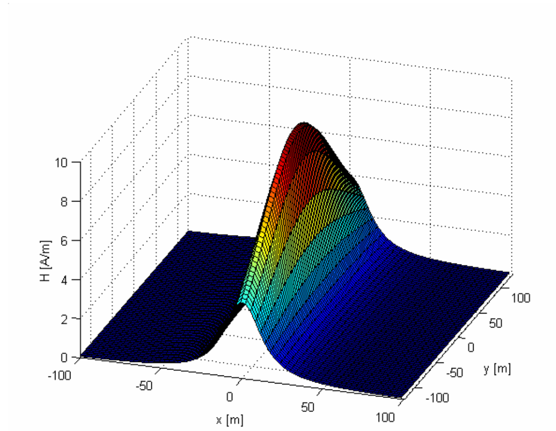

Fig. 11: Total induced magnetic field of 110kV three-phase line at ground (z=0).

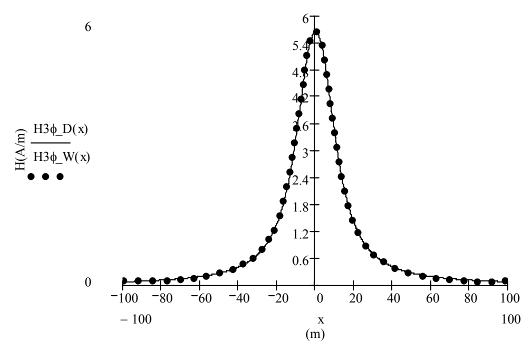

Fig. 12: Comparison betwen two induced magnetic fields of 110kV three-phase line at ground (z=0) above two-layer earth.

Table. 2. presents the comparison of total induced magnetic field values above lossy ground , where soil resistivities are measured by Wenner and Driven Rod methods. Results have seen that error is small about 1-2%, so we can choose one of soil resistivies for calculation of induced magnetic field of the line.

Table. 2. The comparison between $H_{3\phi\_D}$ and $H_{3\phi\_W}$.

X (m)	$H_{3\phi\_D}$	$H_{3\phi\_W}$
0	5.642	5.666
2	5.521	5.542
4	5.166	5.183
6	4.622	4.636
8	3.985	3.996
10	3.353	3.361
15	2.114	2.118
20	1.376	1.377

6. CONCLUSION

This paper presented five different calculation methods of the series impedance matrix of three-phase transmission lines taken into account the complex resistivity, skin effect of earth. In particular, authors proposed the simple formula for calculating the height of the overhead transmission lines in a span between two poles and new analytical formulae of induced magnetic fields calculation of transmission lines, in which taken into account the influence of frequency-dependent earth resistivity and the sag of the line. This method can be applied to all power transmission lines as: medium-voltage; high-voltage; super-high voltage and extra-high voltage lines.

Finally, these calcucation results will be used to consider the influence of magnetic field of power transmission lines to low-voltage equipments, pipelines, communication equipments and human...in the next work.

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