MINIMAL APPROXIMATION AND SH-APPROXIMATION OF SEMIGROUPS RESPECT TO VARIOUS PREDICATES.

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TÓM TẮT

Chúng tôi nghiên cứu về vấn đề xấp xỉ và SH-xấp xỉ của nửa nhóm đối với các mệnh đề khác nhau. Đối với một tập hợp các nửa nhóm có cùng một tính chất cho trước, chúng tôi tìm ra được một nửa nhóm nhỏ nhất đóng vai trò là nửa nhóm xấp xỉ ứng với từng mệnh đề khác nhau: mệnh đề "bằng nhau"; mệnh đề "chứa một phần tử trong nửa nhóm con"; mệnh đề "chứa một phần tử trong nhóm con"; mệnh đề "chia hết", mệnh đề "Green R-, L-, D-, H-tương đương",... Các kết quả chính có thể tìm thấy trong tài liệu tham khảo.

ABSTRACT

Common concept about the approximation of algebraic system was given in research of academician Mansev A.I. "About homomorphisms on finite semigroups". In mentioned work Mansev demonstrated the connection bettween the finite approximation of algebraic system respect to given predicate and algorithm of decidability problems of this predicate in considered system.

We consider a problem of approximation and SH-approximation of semigroup respect to diffirent predicates. For given class of semigroups we try to find out a minimal semigroup approximation respect to considered predicate such as predicate "equality"; predicate "belonging of an element to a subsemigroup"; predicate "belonging of an element to a subgroup"; predicate "dividibility"; predicate "Green L-,R-,H-,D-equivalency",...

I. INTRODUCTION.

A problem of approximation of semigroups is researched by many specialists. They consider an approximation of a class of semigroups A by homomorphisms from A into a semigroup B respect to various predicates. In case, which the semigroup B is infinite, actualy, there exist a problem of finding an minimal semigroup approximation, that is a semigroup, any proper subsemigroup of which cannot become a semigroup approximation for given class of semigroups. There are many examples, which show negative answer to formulated problem. For example, let $B =< a >$ be an infinite cyclic group , generated by an natural number a, then any proper subsemigroup B_1 of B is isomorphic with B. Consequently, if a class K of semigroups is approximable (SH-approximable) by homomorphisms into B, then K is also approximable (SH-approximable) into any proper subgroup of B. Because a set of subgroups of B is an infinite decreasing chain, so we cannot find out a minimal semigroup approximation in this situation. Main results are in [1]-[5].

II. DEFINITIONS.

Let K be a class of semigroups. Let A and B be a semigroups, Φ is the set of all homomorphisms of A into B, P is a predicate defined on pair (A_1, A_2) of subsets A_1, A_2 from A and from $\varphi(A)$ for every $\varphi \in \Phi$.

Definition 1.

A semigroup A is said to be approximable by homomorphisms from Φ with respect to P if and only if, given two subsets A_1, A_2 from A such as $P(A_1, A_2)$ is false, there exists $\varphi \in \Phi$ such as $P(\varphi(A_1), \varphi(A_2))$ is also false.

Definition 2. A semigroup A is said to be SH-approximable by homomorphims from Φ with respect to P if only if, every homomorphic image of any subsemigroup of A is approximable respect to P.

Definition 3. A semigroup B is called a minimal weakly approximation (weakly SH-approximation) semigroup for a class K with respect to P if and only if the following conditions hold:

(i) Any semigroup $A \in K$ is approximable (SH-approximable) with respect to P by homomorphisms into B;

(ii) If a commutative semigroup S is approximable (SH-approximable) with respect to P by homomorphisms into B, then $S \in K$.

(iii) If B_1 is a proper subsemigroup of B, then there exists a semigroup $A_1 \in K$ such as A_1 is not approximable (SH-approximable) with respect to P by homomorphisms into B_1.

Definition 4. A relation ρ, defined on a semigroup A in the following way: $(\forall a, b \in A) \quad (a \rho b \Leftrightarrow aba = a \wedge bab = b)$, is called relation regular contingency. Two elements $a, b \in A$, such as $a \rho b$ are called regular conjugated.

Definition 5. Let Q denote the set of all primes. Let G_p be a quasi-cyclic group of the type p^∞ with identity e_p and with an operation denoted by $\otimes_p$. Put $C^* = \bigcup_{p \in Q} G_p$. Define in C^* multiplication as follows. $\forall a_p, a_q \in C^* : a_p * a_q =$

i) $a_{p \otimes_p a_q}$, if $p = q$;

ii) $a_{\max\{p,q\}}$, if $p \neq q$ and $\max\{p, q\} > 3$;

iii) e_5, if $p \neq q$ and $\max\{p, q\} = 3$.

Direct calculation shows that $C^* = (C^*, *)$ is a semigroup, a semilattice of groups $G_p, p \in Q$.

Another terns or definitions A are in [6].

III. Results.

Theorem 1. A semigroup A is SH-approximable by homomorphisms from A into the semigroup C^* respect to Green R-equivalency if and only if A is a periodic completelly regular semigroup.

Theorem 2. Assume K is a class of semigroups A satisfying the condition:

if an archimed component A_ξ of semigroup contains an idempotent e, then for any two elements $a, b \in A$, such as a and b are not regular conjugated, satisfying either $a^2bz \neq az$ or $ab^2z \neq bz$ for any $z \in A_\xi$.

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