HIGHER-ORDER FDTD, FE AND MESHFREE METHODS FOR SELECTED EMC PROBLEMS IN POWER SYSTEMS: PART I – CALCULATING LIGHTNING-INDUCED VOLTAGES ON TRANSMISION LINES

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ABSTRACT

The Finite-Difference Time-Domain (FDTD) solutions of lightning-induced voltages due to nearby lightning return stroke on overhead power lines are proposed by Agrawal *et al.* [9] and Paolone *et al.* [10]. In this paper, we develop above FDTD method using the higher-order FDTD scheme with second-order accuracy in the time and 18^{th}-order in the space. In particular, 18^{th}-order Finite Element Method (FEM) and a kind of Meshfree Methods, Smooth Particle Hydrodynamics (SPH) method, are also presented and used successfully in this problem. Finally, three numerical methods are applied to calculate lightning-induced voltages on multiconductor distribution lines by solving Field-to-transmission line coupling equations. The solutions of various numerical methods are illustrated and compared by means of graphs. Obtained results are shown that the influences of the phase-conductor position, ground wires and the higher-order FDTD, FE schemes on the peak value of total induced voltages at the first few microseconds are considerable. These algorithms are very suitable for solving telegraphy or EM coupling equations of transmission lines in EMC problems with more accuracy.

Keywords: HO-FD, HO-FE methods, Meshfree Method, multiconductor lines, lightning-induced voltages, EMC.

1. BRIEF INTRODUCTION TO EMC PROBLEMS IN POWER SYSTEMS

Electromagnetic Compatibility (EMC) problems have been investigated by the electromagnetic (EM) systems modeling [1]-[3]. Most fundamental studying of the EMC behavior of electrical-electronics devices and power systems are computed based on Maxwell's and EM coupling equations in the basic concepts of circuit theory and electromagnetics through Kirchoff's Voltage Law and Kirchoff's Current Law arise from Faraday's Law and Ampere's Law, respectively. It's solutions can be obtained using analytical method and numerical methods of an approximate analysis technique in frequency and time domains. During past years, the numerical methods have successfully used in many engineering to avoid the complexity of problems then obtain the accurate solutions.

In power systems engineering, for considering the design of power transmission systems, we must have accurate knowledge of EMC problems and its effects. EMC calculation software is applied more widely in electrical-electronics industry all over the world. With the development of computer and mathematics, numerical methods are increasingly replacing to analytical one for considering EMC problems, where it cannot obtain higher exact solutions.

Introducing the EMC problems in power systems, we illustrate four main groups as follows

- First group is EMC of transmission line systems excited by external fields such as lightning, nuclear detonation, antenna sources and Electromagnetic Interference (EMI) between pipelines and nearby high voltage power lines. In where, we focus on the numerical analysis of induced voltages on transmission lines excited by external field sources or on water and gas pipelines excited by nearby power lines.

- Second one is transient EMI in substation. Focus on the calculation of transient EM fields and EM disturbances coupled in secondary cables due to switching operation in air-insulation-substation (AIS) and gas-insulation-substation (GIS).
- Third one is power systems grounding. We focus on considering grounding system responses of transmission lines or substation of 110kV, 220kV, 500kV and 750kV. The numerical methods are used to calculate current and voltage field distributions in earth, from where the step and touch voltages are obtained. The optimization design of ground grid is also investigated.
- Fourth one is the computation of 2D and 3D EM fields of transmission lines and towers. We deal with analytical and numerical solutions of EM fields around the lines or steel towers of single- and double-circuit three-phase lines.

In our studies, selected first EMC problem in power systems is the electromagnetic coupling between lightning fields and transmission systems, where the lightning-induced voltages on the single and double-circuit three-phase transmission lines are calculated and discussed in Part I. The selected second one is grounding problem of high voltage power lines and substations, where based on numerical computation of current and voltage field distribution in earth, the ground-potential-rise distribution on the grounding grid is calculated in Part II. Further, an optimized design of the grounding grid may be performed.

The choice of these problems is due to the availability of reference results that permit the precision of the numerical solutions using different numerical methods. These problems are still considering and publishing on IEEE transaction on EMC or the proceedings of International Conference of Lightning Protection (ICLP) and International Conference on Power Systems Transient (IPST) and some other conferences of power systems…

2. INTRODUCTION

The lightning field induction on distribution lines is one of the special electromagnetic compatibility problems in power systems. Many authors have investigated and made deeply insight knowledge into the physical bases of this problem such as lightning source, the effect of lightning current shapes, corona, shielding wires, surge arresters, ground transient resistance to the magnitude of lightning-induced voltages and so on [4]-[12]. In most studies, the lightning-induced voltages have been computed based on solving field-to-transmission line coupling equations of distribution lines in the time domain. There are many methods used in this calculation such as analytical methods [4]-[6], numerical methods of Wavelet [7] and FDTD [8]-[12]. The first-order FDTD technique is proposed by Agrawal *et al*. [9] and 2^{nd}-order one proposed by Paolone *et al*. [10]. This has been developed with higher-order FDTD scheme by authors in [11]-[12].

The FDTD techniques are popular and suitable for studying the transient responses of transmission lines in time domain by discretizing their partial differential equations along the line in the space and the time. However, the obtained results have had considerable errors because most authors have solved these equations using standard FDTD methods with second-order accuracy in the time and second-order in the space. Hence it is important to increase the accuracy of the solutions when using the FDTD methods. The higher accuracy of the finite difference solutions is controlled by both two ways as 1) to decrease the size of the space-time integration steps, and 2) to increase the order of finite difference approximations (FDAs) of the derivatives. The first way is easily carried out in the calculation program. The second one are developed and formulated by authors in [12].

The new approach of the Meshfree Method of SPH [14]-[16] is presented in this paper. The Meshfree Method was originated about twenty years ago and has been applied in electromagnetic field and transient computations during few past years, [17]-[21].

This is the first time authors have applied this method to compute lighting-induced voltages on lines. In appendix, we have presented formulae of lightning electromagnetic field calculation using Mesfree Method of SPH.

In this work, we will only focus on the application of three numerical methods to calculate lightning-induced voltages. We first present formulae of the HO-FDTD, HO-FE and Meshfree approximations of first- and second-order partial derivatives of a function *u(x,t)*. We secondly apply these methods to calculate lightning-induced voltages on the vertically- and horizontally-configured single- and double-circuit three-phase distribution lines. The results obtained by using various methods are illustrated and discussed. The influences of electrical strength of distribution line and ground wires to peak voltage values are also discussed.

From results are compared in the theory [12] and in the computations, it has concluded that the solutions of HO-FD and HO-FE Methods are identical at each respective order. The 4^{th}-order FD and FE solutions are equal to the one of the Meshlfree Method. Results are also seen that the presence of ground wires significantly reduced the peak value of lighting-induced voltages.

3. HIGHER-ORDER FINITE DIFFERENCE METHOD

For finding the solution of partial differential equations using FDTD methods, the first step is the whole solution domain in the *x-t* plane must be divided into equal grids of steps Δx and Δt that is called the finite difference grid. The next one, we must determine the FDAs of spatial-temporal derivatives of the function $f(x,t)$ at each grid point in whole grid. In [10], authors developed and formulated higher-order centered-difference approximations of FDTD. We again illustrate general formulae of FDTD as follows

The second-order centered-difference approximation of the first-order temporal derivative of the function $u(x,t)$ at the grid point (k,n) is written as

$$\frac{\partial}{\partial t}u(x,t) = \frac{u_k^{n+1} - u_k^{n-1}}{2\Delta t}. \qquad (1)$$

and the centered-difference approximations of the first- and second-order partial derivatives of the *2M*th-order accuracy in space (*M=1÷9*) are given by

$$\frac{\partial}{\partial x}u = \frac{1}{\Delta x}\left(\sum_{l=1}^{M}(-1)^l A_l\left(u_{k-l}^n - u_{k+l}^n\right)\right), \qquad (2)$$

$$\frac{\partial^2}{\partial x^2}u = -\frac{1}{\Delta x^2}\left(B_0.u_k^n + \sum_{l=1}^{M}(-1)^l B_l\left(u_{k-l}^n + u_{k+l}^n\right)\right). \qquad (3)$$

Note that the coefficients A_l and B_l according to each order of FD approximations in the equations (2)-(3) have been presented in [12].

4. HIGHER-ORDER FINITE ELEMENT METHOD

A. Interpolation functions

Let us consider FEM. Finding the shape functions is a very important step in the FE analysis. We will use the shape function of $\theta_j(x)$ that is the interpolation polynomial of order p_i and is expressed as follows - [13]

$$\theta_j(x) = \frac{\prod_{1\le i\le p_i+1,\, j\ne i}(x - x_i)}{\prod_{1\le i\le p_i+1,\, j\ne i}(x_j - x_i)}. \qquad (4)$$

where:p_i is order of interpolation polynomial, x_i is the coordinate of node *I*, x_j is the coordinate of node *j, x* is the coordinate of a point between nodes *i* and *j*.

B. Finite element discretization

In FE analysis, the first step is to divide whole solution domain into subdivisions that are called elements. Each element has points called interpolation nodes. The second is to find the value of the function at each element based on the shape functions (4). This allows the function $u(x,t)$ to be written in the form of p_ith-order FE approximation as

$$u(x,t) = \sum_{j=1}^{p_i+1}\theta_j(x)u_j^n. \qquad (5)$$

Differentiating both sides of equation (5) with respect to the *x* variable, the first- and second-order partial derivatives of the function $u(x,t)$ are given by

$$\frac{\partial}{\partial x}u = \sum_{j=1}^{p_i+1}\frac{\partial\theta_j(x)}{\partial x}u_j^n, \qquad (6)$$

$$\frac{\partial^2}{\partial x^2}u = \sum_{j=1}^{p_i+1}\frac{\partial^2\theta_j\left(x\right)}{\partial x^2}u_j^n. \qquad (7)$$

In order to find the first- and second-order derivatives of shape functions in the equations (6)-(7), we differentiate both sides of the equation (4) and obtained the equations (8)-(9) as

$$\frac{\partial\theta_j\left(x\right)}{\partial x} = \frac{\sum_{m=1}^{p_i+1}\prod_{\substack{1\le i\le p_i+1\\ j\ne i;\, j\ne m}}\left(x-x_i\right)}{\prod_{1\le i\le p_i+1,\, j\ne i}\left(x_j-x_i\right)}, \qquad (8)$$

$$\frac{\partial^2\theta_j\left(x\right)}{\partial x^2} = \frac{\sum_{\substack{l=1\\ l\ne j}}^{p_i+1}\sum_{\substack{m=1\\ m\ne j\\ m\ne l}}^{p_i+1}\prod_{\substack{1\le i\le p_i+1\\ j\ne i;\, j\ne m\\ j\ne l}}\left(x-x_i\right)}{\prod_{1\le i\le p_i+1,\, j\ne i}\left(x_j-x_i\right)}. \qquad (9)$$

The equations (8)-(9) presented by authors [12], [21] are the general forms of derivatives of shape functions for any order $p_i \ge 2$.

5. MESHFREE METHOD OF SPH

Introducing the Meshfree Method of SPH, we express the approximation of the unknown function $u(x)$ in a domain Ω that consists of a set of nodes x_I with $I = 1 \div n_N$ as in [14]-[16] as follows

$$u^h(x) = \int_\Omega W\left(x-x_I,h\right)u_I d\Omega_I. \qquad (10)$$

where $u^h(x)$ is the approximation. The function $u(x)$ is denoted by u_I at node I and $W(x-x_I,h)$ is fifth-degree Kernel Function [16] that is expressed by (11) and in Fig. 1. as follows

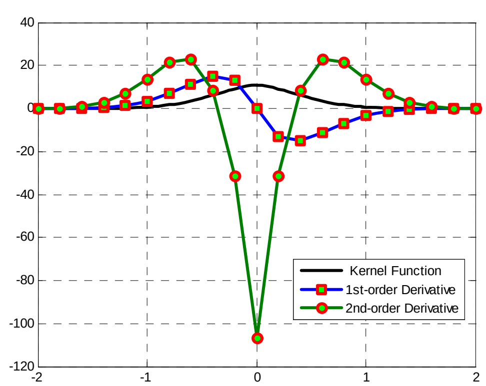


Fig. 1. One-dimensional Kernel function and its derivatives.

$$W(\xi,h) = \frac{C}{h^d}\begin{cases}\left(2-\xi\right)^5 \text{-}16\left(1-\xi\right)^5 & \text{if } \xi < 1\\ \left(2-\xi\right)^5 & \text{if } 1\le\xi\le 2\\ 0 & \text{if } 2\le\xi\end{cases} \qquad (11)$$

with $\xi = \frac{\left\|x-x_I\right\|}{h}$ and $C = \begin{cases}\frac{2}{3} & \text{if } d=1\\ \frac{10}{7\pi} & \text{if } d=2\\ \frac{1}{\pi} & \text{if } d=3\end{cases}$

where: d is the dimension of the problem, h is a measure of the size of the support, also called smooth length.

In the one-dimensional problem, the approximation of the unknown function $u^h(x)$ can be written as in [14]

$$u^h(x) = \sum_I W\left(x-x_I\right)u_I \Delta x_I. \qquad (12)$$

6. NUMERICAL SOLUTION OF LIGHTNING - INDUCED VOLTAGES

A. Field-to-Transmission Line Coupling Equations of MTL

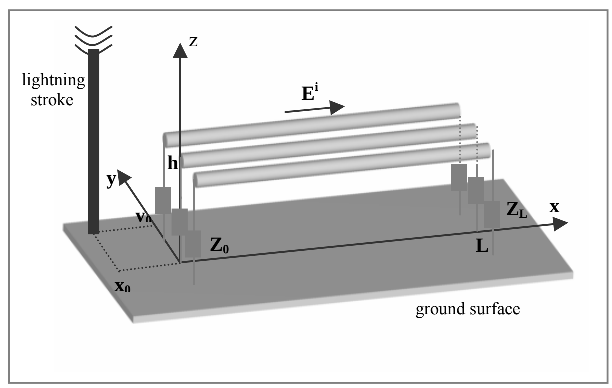


Fig.2. Coordinate system of horizontally-configured MTL excited by lightning return stroke

As in [11]-[12], we use the field-to-transmission line coupling equations proposed by Agrawal *et al.* [9] to calculate lightning-induced voltages on MTL. These equations have been expressed in the matrix form as

$$\frac{\partial\mathbf{V}^s(x,t)}{\partial x} + \mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(x,t) = \mathbf{E}_\mathbf{x}^\mathbf{i}(x,t), \qquad (13)$$

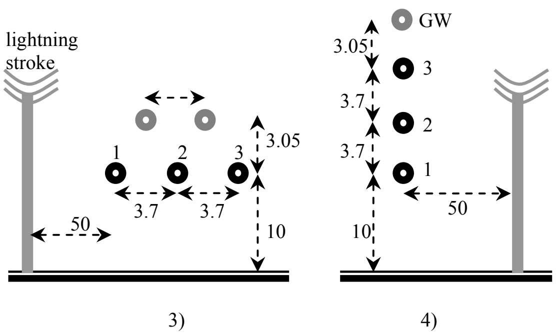
$$\frac{\partial\mathbf{I}(x,t)}{\partial x} + \mathbf{C}\frac{\partial}{\partial t}\mathbf{V}^s(x,t) = 0. \qquad (14)$$

where: **L** and **C** are the inductance and capacitance matrices, all per-unit length. $\mathbf{V}^\mathbf{s}(x,t)$, $\mathbf{I}(x,t)$ are the column vectors of scattered voltages and currents along the lines. $\mathbf{E}^{\mathbf{i}}_{\mathbf{x}}(x,h,t)$ is the column vector of the incident horizontal electric fields along the x axis at the phase-conductor heights in Fig. 2.

Equations (13)-(14) will be solved using HO-FDTD, FE and Meshfree Methods and the results will be presented hereafter.

B. The case of single-circuit three-phase lines

For studying the case of the lightning-induced voltages on MTL, models of horizontally- and vertically-configured three-phase lines with ground wires are used as in Figs. 3.-4. The radius of each conductor is 9.14 mm as in [4], [8]. The height of the line is 10 m. The line length is 1 km. The lightning stroke located is y_0=50m from the line center.



Figs. 3.- 4. Geometrical configuration of horizontally- and vertically-configured three-phase lines

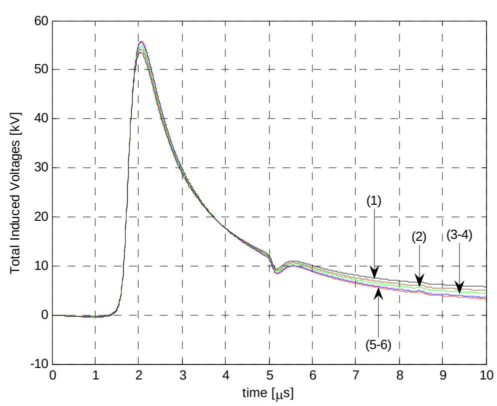


Fig. 5. Comparison between HO-FDTD and HO-FE solutions of total induced voltages on the lossless line at the line termination of conductor 1 in Fig. 3.

where: curve 1 is the solution of (2,8)FDTD and 8th-order FE,…, and curve 6 is the solution of (2,18)FDTD and 18th-order FE.

For considering lighting-induced voltages on distribution lines without ground wires is illustrated on Figs. 3.-.4. Fig.5. shows the comparison between the various HO-FDTD and HO-FE solutions of 8th÷18th order. Induced voltage value will have highest accuracy when we use 18th-order FD, FE schemes. Results have been shown that the solutions solved by HO-FDTD and HO-FE methods are the same. It has been seen that the solution of 16th-order scheme and the one of 18th-order scheme are similar and therefore using more higher-order schemes is not necessary. Notice that we computed all induced voltage values on MTL for configurations of Figs. 3.-.4. and Fig. 11. using 18th-order FDTD and FE Methods.

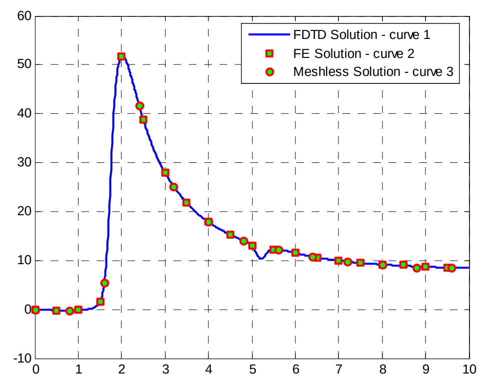


Fig. 6. Comparison between HO-FDTD, HO-FE and Meshfree solutions of total induced voltages on the lossless line at the line termination of conductor 1 in Fig. 3.

Fig. 6. shows the results solved by the 4th-order FDTD, FE and Meshfree Methods. It is seen that the errors between these methods are very small. Therefore we can choose one of three methods for investigating this problem.

On Figs. 7–8. Curve 3 is the scattered voltage computed by field-to-transmission coupling equations of the line adding boundary conditions at two line terminations, curve 2 is the incident voltage computed by the incident vertical electric field and curve 1 is the total induced voltage that is the sum of curves 2 and 3. From these figures, it can be shown that the total induced voltage reaches the peak value after few microseconds. In this study, the total voltage is equal at two ends of the line and it reaches the maximum value at the line mid point, can see the same as in [2].

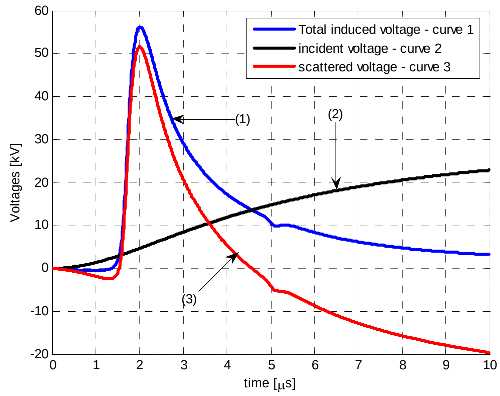


Fig.7. Induced voltage components on the lossless line at x = 0 m.

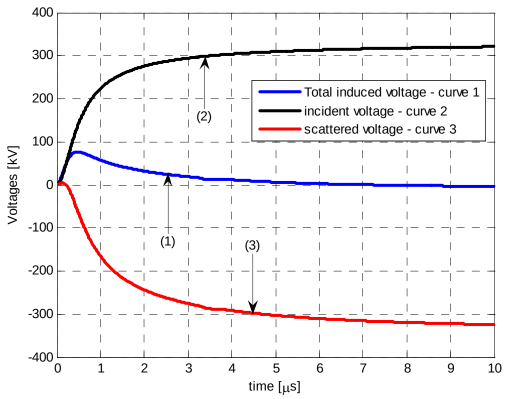


Fig.8. Induced voltage components on the lossless line at x =500 m.

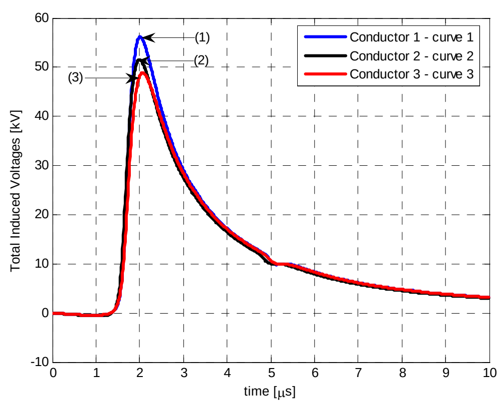


Fig. 9. Total induced voltages on the lossless horizontally-configured three-phase line at the line termination.

Figs. 9.-.10. show the total induced voltages on each conductors of distribution lines as in Figs. 3.-.4. They are computed using FDTD(2,18) and 18th-order FE Methods. It has been again seen that the electrical strength of conductors of lines effected directly to the peak values of voltages. The induced voltage values on conductors 1 and 3 for two configurations are largest. These decrease from the distance between lightning stroke position and conductor in Fig. 3. or from height of conductor in Fig. 4. In two cases, voltage values are smallest on conductors 3 and 1.

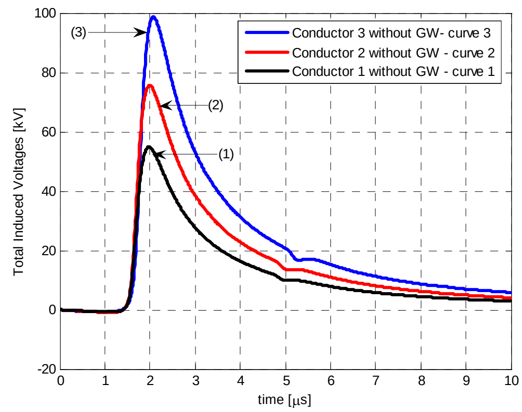


Fig. 10. Total induced voltages on the lossless vertically-configured three-phase line at the line termination.

C. *The case of double-circuit three-phase lines*

For considering the case of double-circuit three-phase line is illustrated as in Fig. 11., we calculate lightning-induced voltages on each conductor of lines and present in Fig. 12.

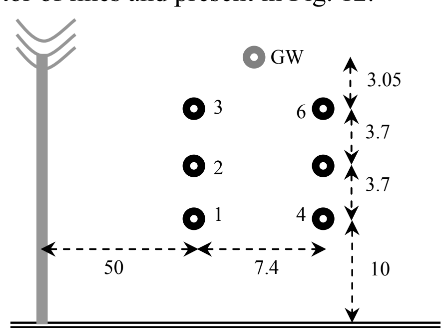


Fig. 11. Geometry model of double-circuit vertically-configured three-phase lines.

Fig. 12. is the general result, it again shows the effect of conductor height is greater than the one of the distance between the lighting stroke and the conductor. Therefore, the design of horizontal lines is better than the vertical one for the reduction of lightning effects.

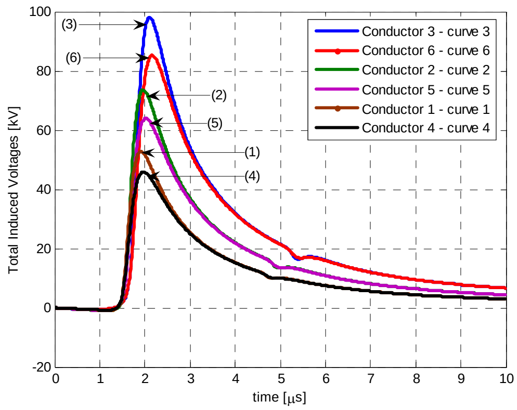


Fig. 12. Total induced voltages on the lossless double-circuit vertically-configured three-phase line at the line termination.

7. INFLUENCE OF GROUND WIRES TO LIGHTNING-INDUCED VOLTAGES

For investigating the effect of ground wires on induced voltage value, we deal with the presence of GW for the same three-phase lines as in Figs. 3.-.4. The position of GW is the same as in [4], [8].

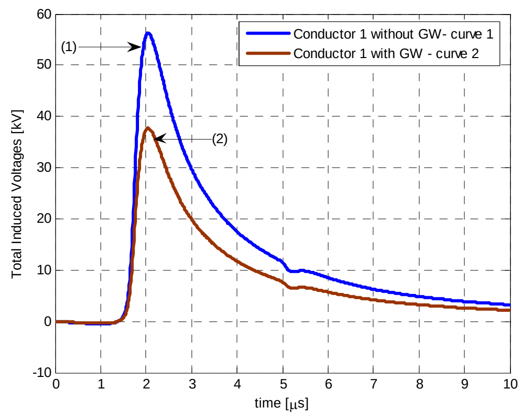


Fig. 13. Total induced voltages on conductor 1 of lines with and without ground wire in Fig. 3.

Fig. 13. shows induced voltage values on conductor 1 of Fig. 3. are computed in two cases of lines with and without GW. It is seen that, the presence of GW significantly reduced the peak voltage value and the protective ratio can be calculated by simulation result or by Ruck's simplified formula is again presented in [8].

Fig. 14. shows the effect of GW on peak voltage values of three conductors of lines in Fig. 4. The effective ratio depends on the distance between the ground wire and the conductor, so this ratio will be different for each conductor. It can be also calculated by Suck's formula.

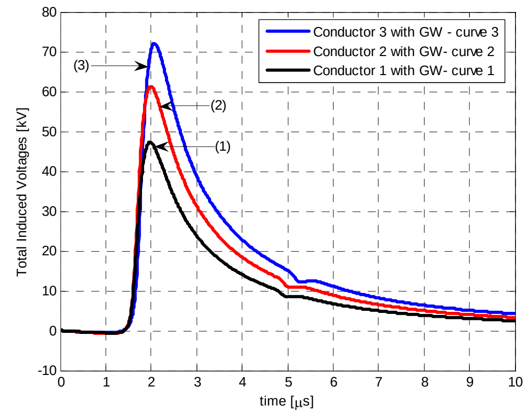


Fig. 14. Total induced voltages on three conductors of lines with ground wire in Fig.4.

8. CONCLUSION

In this paper, authors have formulated and presented general formulae of HO-FD and HO-FE approximations. Additionally, authors have for the first time successfully applied the HO-FE and Meshfree Methods to compute lightning-induced voltages on MTL. Results have shown the equivalence between the solutions of HO-FDTD, HO-FE and Meshfree Methods. The calculation results concluded that the accuracy of solution is increased using higher-order approximations and the presence of ground wires reduced lightning-induced voltages on lines.

In particular, the algorithm and general formulae of HO-FD and HO-FE approximations can help readers to easily formulate p_ith-order FD and FE approximations of $p_i \geq 18$ and apply them to investigate many Electromagnetic Field and EMC problems with more accuracy.

Finally, this development of mathematics will be widely used in other engineering, mechanics, civil…in the future.

7. APPENDIX

In this appendix we present the numerical algorithm of lightning electromagnetic field computation using a combination between FDTD and Meshfree Methods.

Mawell's equations are expressed in the cylindrical coordinates using numerical method as follows

$$\left(E_r\right)_R^{n+1} = \frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t}\left(E_r\right)_R^n - \frac{2\Delta t}{2\varepsilon + \sigma\Delta t}\sum_{R'=1}^{I}\left(H_\varphi\right)_{R'}^{n+\frac{1}{2}} \frac{\partial W(R-R')}{\partial(r-r')}\Delta V_I. \quad (15)$$

where σ is ground conductivity, ε is ground permittivity, W is Kernel function (11) in 2D domain and ΔV_I is measure of the domain surrounding node I.

Note that the components $H\varphi$ and E_z are also formulated the same as (15) using FDTD-Meshfree methods.

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