SIMPLE NONLINEAR CONTROL FOR SMOOTH SEAM TRACKING OF A WELDING MOBILE ROBOT

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ABSTRACT

In this paper, an simple nonlinear controller is applied to two-wheeled welding mobile robot to track a smooth curved welding path. The mobile robot is considered in terms of kinematic model and dynamics model in Cartesian coordinates. The system has nonholomic constraints in relation with its coordinates and the reference welding path. To obtain the controllers, the tracking errors are defined, and the controllers are derived based on backstepping technique to guarantee that the errors converge to zero asymptotically. The touch sensor consists of two potentiometers including one linear potentiometer for measuring distance error, and one rotating potentiometer for measuring angular error. To implement the experiment, a control system is developed based on the integration of three PIC18F452's: two for servo DC motor controllers, and one for main controller. The two servo controllers can perform indirect servo control using one encoder. The main controller which functions as master links to the three servo controllers, as slave, via I2C communication. Also, the simulation and experimental results are included to illustrate the performance of the proposed controller.

1. INTRODUCTION

Automatic control of welding process is widely used in many branches of industry such as building construction, pipelines, aircraft, ship fabrication, automobiles, etc. And one of the most complex applications is welding systems based on mobile robots that can give several benefits: generating a perfect welding movement and producing a consistent weld penetration and weld strength.

Mobile robot under nonholonomic constraints has been attracted much attention of many researchers in literature. Of these researches, some adaptive feedback controllers have been proposed to solve the tracking problem in terms of the dynamics model. T. Fukao, 2000, proposed the integration of a kinematic controller and a torque controller for the dynamic model of a nonholonomic mobile robot. In this design, a kinematics adaptive tracking controller is proposed, and then a torque adaptive controller with unknown parameters is derived using the kinematic controller [1]. T. H. Bui et al., 2003, proposed the adaptive nonlinear controller for two-wheeled welding mobile robot tracking a smooth-curved welding path using Lyapunov function candidate with the unknown parameter of moment of inertia [2]. Fierro, 1995,

developed a combined kinematic and torque control law using backstepping approach, and the asymptotic stability is guaranteed by Lyapunov [3]. In this design, the parameters of the mobile robot are necessary, which is almost impossible to obtain exactly in practice: it is needed to be estimated. And this is the case of this paper.

In this paper, an simple nonlinear controller is applied to two-wheeled welding mobile robot tracking smooth-curved welding path in the kinematic and dynamic models based on backstepping technique. The stability is proven using the Lyapunov method. To design the tracking controller, the errors are defined between the welding point on torch and the reference point moving at a specified constant speed on welding path. The kinematic velocity to make the errors to approach zeros as reasonable as desired for practical application. The control law is extracted from the stable conditions respectively. To realize the above controllers, a simple way for sensing the errors using potentiometers is introduced To implement the experiment, a control system is developed based on the integration of three PIC18F452's: two for servo DC motor controllers, and one for main controller. The two servo controllers can perform indirect servo control using one encoder. The main controller which

functions as master links to the three servo controllers, as slave, via I2C communication. Additionally, the simulation and experimental results have been done to show the effectiveness of the proposed controller.

2. DYNAMIC MODEL OF WMR

In this section, the dynamic of two-wheeled welding mobile robot is considered with the nonholonomic constraints in relation with its coordinates and the reference welding path.

Fig. 1 WMR model

Welding mobile robot used in this paper is of two-wheel mobile robot with some modifications on mechanical structure for welding application (Fig. 1). It is observable that the mobile platform has nonholonomic constraint; consequently, it makes tracking error which is perpendicular to the mobile robot motion slow to converge, if not impossible in some cases of initial posture.

The model of two-wheeled welding mobile robot is shown in Fig. 2. The posture of the mobile robot can be described by three generalized coordinates:

$$q = \begin{bmatrix} x & y & \phi \end{bmatrix}^T \tag{1}$$

where (x, y) is Cartesian coordinates of the WMR's center and ϕ is the heading angle of the WMR

Also it is chosen the internal state variables as follows

$$z = \begin{bmatrix} v & \omega \end{bmatrix}^T \tag{2}$$

We assume that the wheels roll and do not slip, that is, the robot can only move in the direction normal to the axis of the driving wheels. Analytically, the mobile base satisfies the conditions as the following

$$\dot{y}\cos\phi - \dot{x}\sin\phi = 0 \tag{3}$$

The dynamic model for the above wheeled mobile robot is given as follows [8]

$$\begin{cases} \ddot{x} = \dfrac{\lambda}{m}\sin\phi + b_1 u_1 \cos\phi \\[2mm] \ddot{y} = -\dfrac{\lambda}{m}\cos\phi + b_1 u_1 \cos\phi \\[2mm] \ddot{\phi} = b_2 u_2 \end{cases} \tag{4}$$

where $b_1 = 1/(rm), b_2 = b/(rI)$, and that m and I denote the mass and the moment of inertia of the mobile robot, respectively. Also, $u_1 = T_1 + T_2$ and $u_2 = T_1 - T_2$ are the control inputs, and λ is the Lagrange multiplier, given by $\lambda = -m\dot{\phi}(\dot{x}\cos\phi + \dot{y}\sin\phi)$. The assumption that the signs of b_1 and b_2 are known is practical since b_1 and b_2 represent combinations of the robot's mass, moment of inertia, wheel radius, and distance between the rear wheels constant with known signs.

First, the kinematic equations of the WMR in the Cartesian space corresponding to are set up as the following

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{5}$$

The relationship between ω and the angular velocities of two driving wheels is the following

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} 1/r & b/r \\ 1/r & -b/r \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

where ω_{rw}, ω_{lw} represent the angular velocities of right and left wheels, b is distance from WMR's center point to the driving wheel, r is the radius of wheel.

Second, the welding point $W(x_w, y_w)$ on torch and its orientation angle ϕ_w can be derived from WMR's center $C(x, y)$ as

$$\begin{cases} x_w = x - l\sin\phi \\ y_w = y + l\cos\phi \\ \phi_w = \phi \end{cases} \tag{7}$$
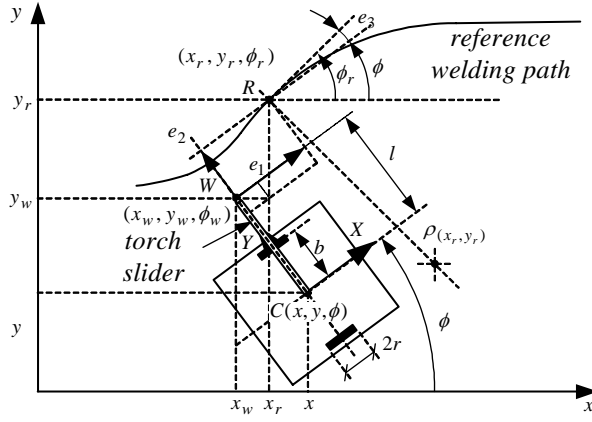
where l is the length of torch

Fig. 2 Scheme for deriving WMR kinematic equations

The derivative of (7) yields

$$
\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{\phi}_w \end{bmatrix} = \begin{bmatrix} \cos\phi & -l\cos\phi \\ \sin\phi & -l\sin\phi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} -\dot{l}\sin\phi \\ \dot{l}\cos\phi \\ 0 \end{bmatrix} \tag{8}
$$

It is assumed that a reference point $R(x_r, y_r)$ on the reference path moving at the constant velocity of v_r with the orientation angle ϕ_r. The dynamic equation is shown below

$$
\begin{cases} \dot{x}_r = v_r \cos\phi_r \\ \dot{y}_r = v_r \sin\phi_r \\ \dot{\phi}_r = \omega_r \end{cases} \tag{9}
$$

where ϕ_r is defined as the angle between v_r and x axis, and ω_r is the rate of angular change of v_r.

3. TRACKING CONTROLLER DESIGN

The scheme of errors are shown in Fig. 2, and the tracking errors $e = [e_1, e_2, e_3]^T$ are defined as the following

$$
e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_w \\ y_r - y_w \\ \phi_r - \phi_w \end{bmatrix} \tag{10}
$$

The first derivative of errors yields

$$
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 \\ \omega_r \end{bmatrix} \tag{11}
$$

We will design a controller to achieve $e_i \to 0$ when $t \to \infty$; in other words, the welding point W tracks to the reference point R at a desired velocity of welding.

Step 1: Kinetic Controller Design

A kinematic controller is designed to achieve $e_i \to 0$ when $t \to \infty$; in other words, the welding point W tracks to the reference point R at a desired welding velocity.

The Lyapunov function candidate is chosen as

$$
V_1 = \frac{1}{2}(e_1^2 + e_2^2) + \frac{1}{k_2}(1 - \cos e_3) \tag{12}
$$

The derivative of V_1 becomes

$$
\dot{V}_1 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + \frac{1}{k_2} \sin e_3 \dot{e}_3
$$
$$
= e_1(-v_d + v_r \cos e_3) + \frac{1}{k_2} \sin e_3 (\omega_r - \omega + k_2 e_2 v_r) \tag{13}
$$

To achieve $\dot{V}_1 \le 0$, the control law called virtual control for the mobile platform is chosen as the following:

$$
\begin{aligned} v = v_d &= v_r \cos e_3 + k_1 e_1 \\ \omega = \omega_d &= \omega_r + k_2 v_r e_2 + k_3 \sin e_3 \end{aligned} \tag{14}
$$

where v_d and ω_d are considered as its desired virtual control variables.

Substituting (14) into (13) yields the following

$$
\dot{V}_1 = -k_1 e_1^2 - \frac{k_3}{k_2} \sin^2 e_3 \le 0 \tag{15}
$$

Clearly, $\dot{V}_1 \le 0$ and the tracking errors $\mathbf{e} = [e_1, e_2, e_3]^T$ is bounded along the system's solution. It is also assumed that not only the velocity of $v_r > 0$ is constant with the orientation angle ϕ_r but also the reference angular velocity ω_r is bounded and have its bounded derivative for all t in case of the welding mobile robot used in this paper, using Eqs. (11) and

(15), it is shown that $\|\mathbf{e}\|$ and $\|\dot{\mathbf{e}}\|$ are bounded, so that $\|\ddot{V_1}\| < \infty$, i.e., $\dot{V_1}$ is uniformly continuous. Since $V_1(t)$ does not increases and converges to some constant value, by Barbalat's lemma, $\dot{V_1} \to 0$ as $t \to \infty$. As $t \to \infty$, the limit of Eq. (15) becomes

$$0 = K_1 K_4 e_1^2 + K_3 e_3^2 \tag{16}$$

Eq. (16) implies that $[e_1 \quad e_3]^T \to 0$ as $t \to \infty$.
From Eq. (11), the derivative of error e_3 is given:

$$\dot{e}_3 = \omega_r - \omega \tag{17}$$

Substituting ω in Eq. (17) by the auxiliary control input ω_d (Eq. (14)), the following is yielded:

$$\dot{e}_3 = -k_2 e_2 v_r - k_3 \sin e_3 \tag{18}$$

Since $e_3 \to 0$ as $t \to \infty$, the limit of Eq. (18) yields

$$\dot{e}_3 = -k_2 e_2 v_r \tag{19}$$

Since equation $v_r^2 e_3$ has the limit equal to zero when $t \to \infty$, the derivative of this equation yields

$$\frac{d}{dt}\left(v_r^2 e_3\right) = -k_2 v_r^3 e_2 \tag{20}$$

The term $v_r^3 e_2$ is uniformly continuous since its time derivative is bounded. From Barbalat's Lemma, $\frac{d}{dt}\left(v_r^2 e_3\right)$ tends to zero. Therefore, $v_r^3 e_2$ tends to zero, and thus $v_r e_2$ tends to zero. Because the velocity of v_r is constant, $e_2 \to 0$ as $t \to \infty$ from Eq. (20). Hence, the equilibrium point $\mathbf{e} = 0$ is uniformly asymptotically stable.

Step 2: Dynamic Controller Design

The v and ω are considered as virtual control variables for the mobile robot. The control objective is that the trajectory tracking error e converges to zero asymptotically and v , ω converge to v_d , ω_d , respectively even in the presence of unknown parameters in the model of the dynamic system. To solve the above problem, $\tilde{v}$ and $\tilde{\omega}$ are defined as virtual velocity control errors:

$$\begin{aligned}\tilde{v} &= v - v_d \\ \tilde{\omega} &= \omega - \omega_d\end{aligned} \tag{21}$$

Total error vector is defined as

$$\boldsymbol{\varepsilon} = [e_1, e_2, e_3, \tilde{v}, \tilde{\omega}]^T \tag{22}$$

The time derivative of the virtual velocity control error can be derived as follows:

$$\begin{aligned}\dot{\tilde{v}} &= \dot{v} - \dot{v}_d = b_1 u_1 - \dot{v}_d \\ \dot{\tilde{\omega}} &= \dot{\omega} - \dot{\omega}_d = b_2 u_2 - \dot{\omega}_d\end{aligned} \tag{23}$$

Proof.
From the nonholonomic constraint (3), the derivative of v is

$$\begin{aligned}\dot{v} &= \ddot{x}\cos\phi - \dot{x}\sin\phi\,\dot{\phi} + \ddot{y}\sin\phi + \dot{y}\cos\phi\,\dot{\phi} \\ &= (\frac{\lambda}{m}\sin\phi + b_1 u\cos\phi)\cos\phi - \dot{x}\sin\phi\,\dot{\phi} + \\ &\quad (-\frac{\lambda}{m}\cos\phi + b_1 u_1\sin\phi)\sin\phi + \dot{y}\cos\phi\,\dot{\phi} \\ &= \frac{\lambda}{m}\sin\phi\cos\phi + b_1 u\cos^2 u - \dot{x}\sin\phi\,\dot{\phi} \\ &\quad - \frac{\lambda}{m}\cos\phi\sin\phi + b_1 u_1\sin^2\phi + \dot{y}\cos\phi\,\dot{\phi} \\ &= b_1 u_1 - (\dot{x}\sin\phi - \dot{y}\cos\phi)\dot{\phi} = b_1 u_1\end{aligned} \tag{24}$$

Similarly,

$$\dot{\omega} = b_2 u_2 \tag{25}$$

where $b_2 = \dfrac{1}{rm} \neq 0$ and $b_2 = \dfrac{b}{rI} \neq 0$.

From Eqs. (13), (14) and (21), the derivative of V_1 is

$$\begin{aligned}\dot{V_1} &= e_1(-v_d - \tilde{v} + v_r\cos e_3) \\ &\quad + \frac{1}{k_2}\sin e_3(\omega_r - \omega_d - \tilde{\omega} + k_2 e_2 v_r) \\ &= -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 - \tilde{v}e_1 - \tilde{\omega}\frac{1}{k_2}\sin e_3\end{aligned} \tag{26}$$

where k_1, k_2 and k_3 are positive constants.

To obtain the dynamic mobile platform control law, a nonlinear feedback control $u \in R^{n-m}$ must be designed so that $v \to v_d$ as $t \to \infty$. It cannot be guaranteed that Eq. (26) has always negative value due to $\tilde{v}$ and $\tilde{\omega}$.

Consider the new Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}(\tilde{v}^2 + \tilde{\omega}^2) \tag{27}$$

The time derivative of V_2 is

$$
\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + \tilde{v}\dot{\tilde{v}} + \tilde{\omega}\dot{\tilde{\omega}} \\
&= -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 + \tilde{v}(-e_1 + b_1 u_1 - \dot{v}_d) \\
&\quad + \tilde{\omega}(-\frac{1}{k_2}\sin e_3 + b_2 u_2 - \dot{\omega}_d)
\end{aligned}
\tag{28}
$$

To achieve $\dot{V}_2 \leq 0$, the following adaptive control law is chosen as:

$$
\begin{aligned}
u_1 &= \frac{1}{b_1}(e_1 - c_1\tilde{v} + \dot{v}_d) \\
u_2 &= \frac{1}{b_2}(-c_2\tilde{\omega} + \frac{1}{k_2}\sin e_3 + \dot{\omega}_d)
\end{aligned}
\tag{29}
$$

where c_1, c_2 are positive constant.

From (27) and (28), the following can be obtained.

$$
\dot{V}_2 = -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 - c_1\tilde{v}^2 - c_2\tilde{\omega}^2 \leq 0 \tag{30}
$$

Since V_2 is lower bounded and $\dot{V}_2$ is negative semi-definite, V_2 converges to a finite limit. Also, $V_2\ e_1, e_2, e_3, \tilde{v}, \tilde{\omega}$ are all bounded.

Furthermore, the second derivative of V_2 can be written as

$$
\begin{aligned}
\ddot{V}_2 &= -2k_1 e_1[(e_2 + l)(\omega_r + k_2 v_r e_2 + k_3 \sin e_3 + \tilde{\omega}) \\
&\quad - (v_r \cos e_3 + k_1 e_1 + \tilde{v}) + v_r \cos e_3] \\
&\quad - 2\frac{k_3}{k_2}\sin e_3 \cos e_3[\omega_r - (\omega_r + k_2 v_r e_2 + k_3 \sin e_3) - \tilde{\omega})] \\
&\quad - 2c_1\tilde{v}(-c_1\tilde{v} + e_1) - 2c_2\tilde{\omega}(-c_2\tilde{\omega} + \frac{1}{k_2}\sin e_3)
\end{aligned}
\tag{31}
$$

With the assumption that v_r, ω_r and their derivatives are bounded and from the above results, $\ddot{V}_2$ is bounded, that is, $\dot{V}_2$ is uniformly continuous. Since $V_2(t)$ is differentiable and converges to some constant value and that $\ddot{V}_2$ is bounded, by Barbalat's lemma, $\dot{V}_2(t) \to 0$ as $t \to \infty$. This, in turn, implies that $e_1, e_3, \tilde{v}$, and $\tilde{\omega}$ converge to zero. Then $e_2 \to 0$ when $t \to \infty$ as the same way of kinematic controller. The equilibrium point $\varepsilon = 0$ is uniformly asymptotically stable.

4. MEASUREMENT OF THE ERRORS

In this paper, the controller is derived from measurement of the tracking errors e_1, e_2, e_3. The errors measurement scheme is described in Fig. 3: the two rollers are placed at O_1 and O_2. The roller at O_1 is used to specify the two errors e_1 and e_2 and the other, error e_3.
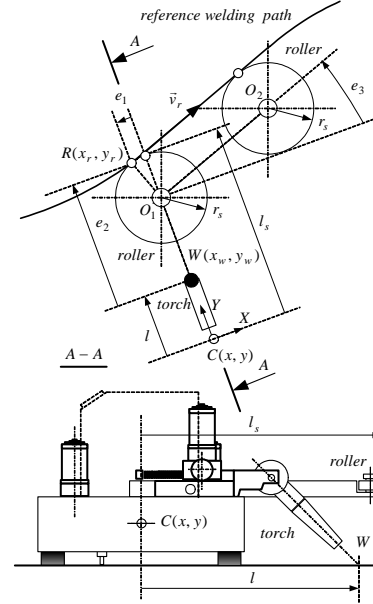

Fig. 3 Scheme for measuring the errors

From Fig. 3, we have the relationships

$$
\begin{cases}
e_1 = -r_s \sin e_3 \\
e_2 = (l_s - l) - r_s(1 - \cos e_3) \\
e_3 = \angle(O_1 C, O_1 O_2) - \pi/2
\end{cases}
\tag{32}
$$

where r_s is the radius of roller, and l_s is the length of sensor. And the two potentiometers are used for measuring the errors: one linear potentiometer for measuring $(l_s - l)$ and one rotating potentiometer, the angular between X coordinate of WMR and $\vec{v}_r$.

5. THE EXPERIMENT DEVELOPMENT AND SIMULATION RESULTS

5.1 The experiment development

The control system was modularized on function to perform special control. The control system is based on the integration of three PIC18F452's: two for servo DC motor controllers, and one for main controller. The two servo controllers can perform indirect servo control using one encoder. The main controller which functions as master links to the three servo controllers, as slave, via I2C communication. The two A/D ports on master are connected to the two potentiometers for

sensing the errors, as mentioned in section 4. The total configuration of the control system is shown in Fig. 4.
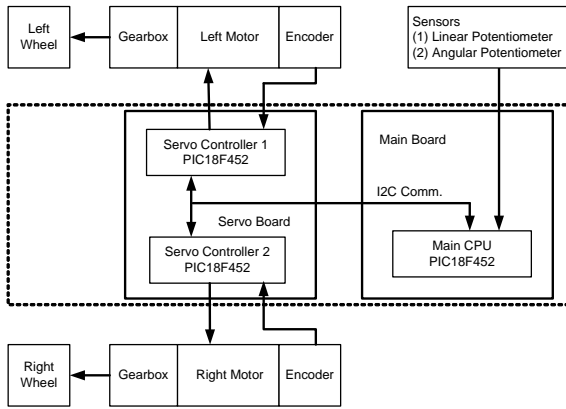

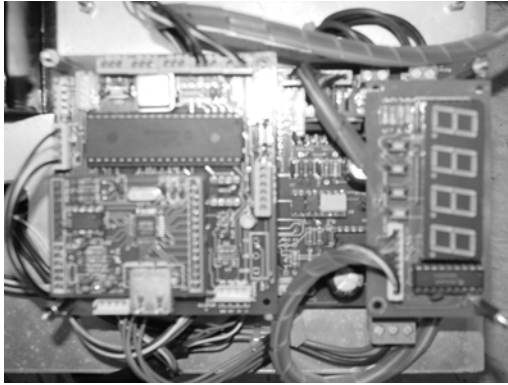Fig. 4 The configuration of the control system
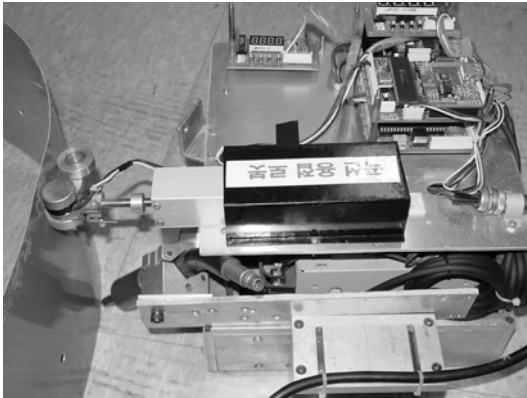

Fig. 5 The controller of the system

Photo. 1 The experimental welding mobile robot

For operation, the main controller receives signals from sensors to achieve the errors, the control laws are rendered based on the errors for the sampling time of 10ms, and send the results commands to the two servo controllers via I2C, respectively. The controller of the system is shown in Fig. 5. The experimental welding mobile robot is shown in Photo 1

5.2 The simulation results

To verify the effectiveness of the proposed controller, simulations have been done with simple nonlinear controller (25) based on backsteping method with a defined reference smooth curved welding path as shown in Fig. 6.

The const parameters are chosen as $c_1 = c_2 = 100$ and $\gamma_1 = \gamma_2 = 10$. The robot's dynamic parameters are chosen as $b_1 = b_2 = 0.5$. The WMR's initial values and parameter values for the simulation are given in Table 1 and Table 2, respectively. The welding speed is 7.5 mm/s.

Table 1. The initial values for the simulation

Parameters	Values	Unit
x_r	0.28	m
x_w	0.270	m
y_r	0.4	m
y_w	0.390	m
ϕ_r	20	deg.
ϕ_w	0	rad/s
v	0	m/s
ω	0	rad/s
l	0.24	m

Table 2. The parameter values for the simulation

Parameter	Value	Parameter	Value
$b\,[\,m\,]$	0.105	$d\,[\,m\,]$	0
$r\,[\,m\,]$	0.025	$m_c\,[\,kg\,]$	16.9
$m_w\,[\,kg\,]$	0.3	I_c [kgm2]	0.2081
I_w [kgm2]	0	I_m [kgm2]	4.96×10^{-4}

The simulation results are given through Figs. 7-13. In Fig. 7, it can be seen that the errors go to zeros after 5 seconds. The virtual linear velocity input v_d for kinematic controller and real linear velocity v for dynamic controller are shown in Fig. 8 for kinematic controller. The virtual angular velocity input ω_d for kinematic controller and real angular velocity ω for dynamic controller are shown in Fig. 9 for kinematic controller. The error of virtual velocity control error $\tilde{v}$ and $\tilde{\omega}$ approaches zero in Fig. 10 after 10 seconds. Fig. 11 show control inputs u_1 and u_2. Fig. 12 shows torques T_1 and T_2 of right and left wheels of MWR, respectively. Fig. 13 shows angular velocities (rpm) of right and left wheels of WMR The posture and the welding trajectory are shown in Fig. 13.
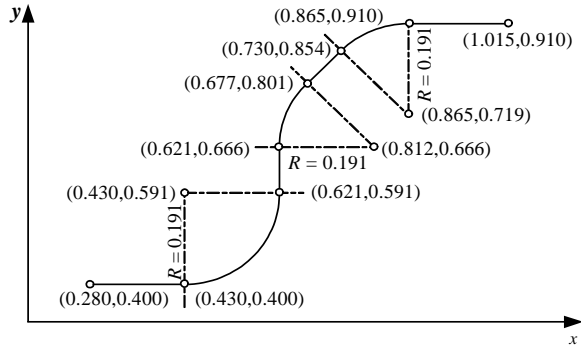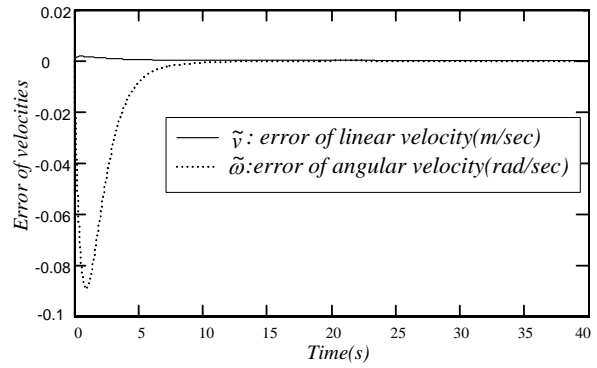
Fig. 6 The reference welding path

Fig. 10 virtual velocity control error $\tilde{v}$ and $\tilde{\omega}$
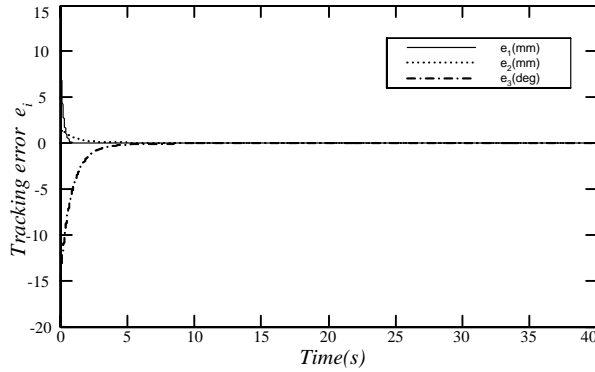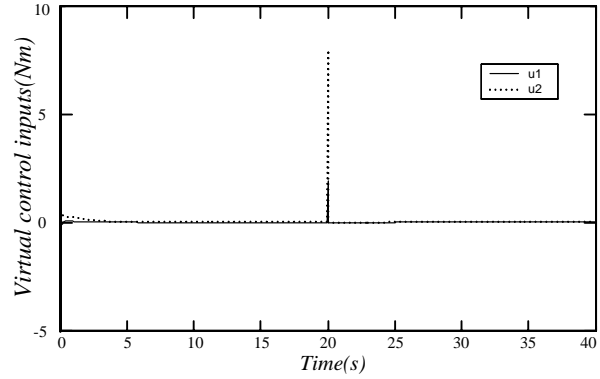

Fig. 7 Tracking errors

Fig. 11 Control input u (5s)

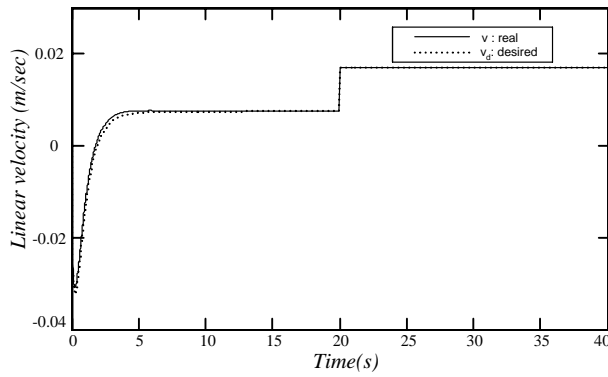

Fig. 8 Linear velocity
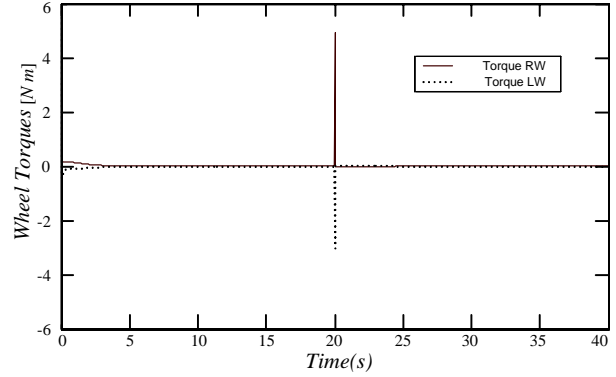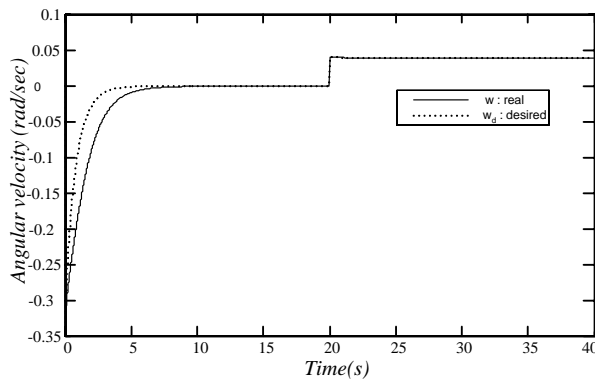

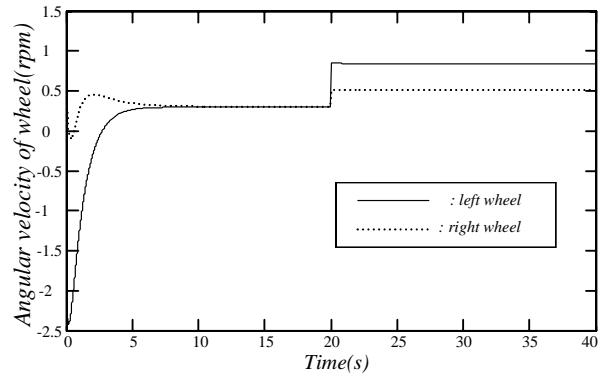Fig. 12 Torques

Fig. 9 Angular velocity.
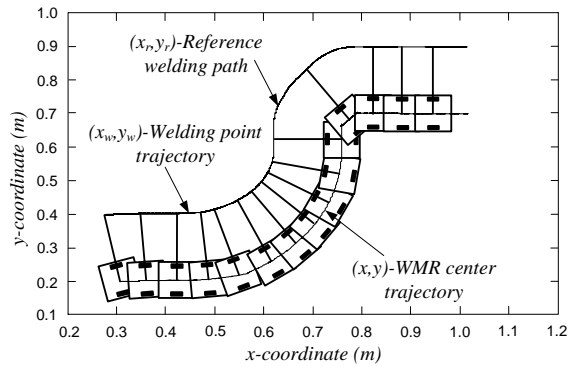

Fig. 13 Angular velocity of wheels

Fig. 13 WMR's movement tracking reference path

6. CONCLUSIONS

In this paper, a simple nonlinear controller is proposed for welding mobile robot to track smooth curved welding path. The controller is designed for the mobile robot considered in kinematic and dynamic models based on backstepping method using Lyapunov function. To design the controller, an error configuration is defined Also, a simple way of measuring the errors for deriving the control law using two potentiometers was proposed. The effectiveness of the proposed controller is proven through the simulation and experimental results. It is shown that the controller is possible and applicable in the practical welding mobile robot. This control system of the proposed controller is simpler than that proposed by Bui. three PIC18F452's for this controller and four PIC16F877's for Bui's controller are needed because one PIC for controlling torch slider in this proposed control system is not needed.

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