PREDICTIVE CONTROL FOR SHAPE MEMORY ALLOY ACTUATORS

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ABSTRACT

Shape Memory Alloy (SMA) actuators which have ability to return to a predetermined shape when heated have many potential applications in aeronautics, surgical tools, robotics, and so on. Although the number of applications is increasing, there has been limited success in precise motion control of these smart actuators. This paper presents a new development of SMA position control system by using Generalized Predictive Control (GPC) algorithm. The use of GPC controller is to generate a control sequence by minimizing a cost function in such a way that the future system output is driven close to reference over finite prediction horizon. The experimental results from real time control using GPC method compared with conventional PID controller are also shown in this paper.

Key Words: SMA, actuators, GPC, identification, real time.

1. INTRODUCTION

SMA is smart material which exhibits the shape memory effect property. This occurs through a solid-state phase change by molecular rearrangement. The two phases, which occur in SMA, are martensite and austenite. As higher temperatures, the material is in the austenite phase. As the temperature is lowered, material changes to martensite phase and grows until at sufficiently low temperatures. This unusual characteristic of SMA actuators has a wide variety of applications in control systems beside conventional types such as electric, hydraulic, and pneumatic actuators.

Model – based control for SMA actuators has been presented in only a few reports. Usually Preisach model of hysteresis based on phenomenological nature [2], [7] is developed. Although the Preisach model has been found widespread acceptance to capture the major features of hysteresis phenomena, there have been some restrictions on the accuracy and computation time the inversion of this model. These are caused by the limitation of switching points in Preisach plane, the inaccuracy and time consuming of data collection from first order transition curves. This paper presents a new development of SMA position control system by using generalized predictive control strategy. Since Generalized Predictive Control (GPC) [3], [5], [10] has been known as an effective tool for the self – turning control of many practical processes, therefore it is suitable to be applied in slow response SMA systems. The use of GPC algorithm is to generate control sequence by minimizing a cost function in such a way that the future system output is driven close to reference over finite prediction horizon. Hence this control strategy can be used to compensate the hysteresis effect and improve accuracy for the displacement of shape memory alloy actuators.

The remainder of the paper is organized as follows. Section 2 provides a briefly review of the on –line model identification method by using recursive parameter estimation algorithm [1], [6]. The real time implementation of GPC algorithm is presented in section 3. In section 4, experimental results for SMA position control system obtained from GPC algorithm are compared with conventional PID control. Concluding rermarks are provided in section 5.

2.RECURSIVE MODEL IDENTIFICATION

Adaptive control is mostly based on the on – line system identification. In this section, non – linear model can be approximately linearized around a particular operating point and written in the form

$$y(t)=\varphi^T(t)\theta+\varepsilon(t) \quad (1)$$

where $y(t)$ is the output at time t, $\varphi(t)=[\varphi_1(t)\ \varphi_2(t)...\varphi_n(t)]^T$ contains known information at time t, for instance old output and input signals from dynamic system, $\theta=[\theta_1\ \theta_2\ ...\theta_n]^T$ consists of n parameters that we want to estimate based on available information, $\varepsilon(t)$ is denoted as model error.

As already known, the objective is to fit the parameter vector θ in model (1), such that the equation error ε(t) gets as small as possible. A following least squares criterion has been used to measure how well the model fits the experimental data:

$$J(\theta)=\frac{1}{2}\sum_{t=1}^{N}\varepsilon^2(t) \quad (2)$$

the minimum of this criterion is obtained when

$$\frac{dJ(\theta)}{d\theta}=\sum_{t=1}^{N}\frac{d\varepsilon(t)}{d\theta}\varepsilon(t)=0 \quad (3)$$

Using equation (1), we obtain:

$$\frac{d\varepsilon(t)}{d\theta}=\frac{d}{d\theta}\left[y(t)-\varphi^T(t)\theta\right]=-\varphi(t) \quad (4)$$

Note that the output $y(t)$ and the regression vector $\varphi(t)$ consist of information from real process, and therefore do not depend on the parameter vector θ in the considered regression model. From equations (3) and (4), the parameter θ is determined by the relation:

$$\sum_{t=1}^{N}\varphi(t)\varepsilon(t)=0 \quad (5)$$

To get the final solution, from (1) and (5), we have:

$$\sum_{t=1}^{N}\varphi(t)\left[y(t)-\varphi^T(t)\theta\right]=\sum_{t=1}^{N}\varphi(t)y(t)-\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\theta=0$$

therefore $\left[\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]\hat{\theta}_N=\sum_{t=1}^{N}\varphi(t)y(t)$

which gives the estimation of parameter θ:

$$\hat{\theta}_N=\left[\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\sum_{t=1}^{N}\varphi(t)y(t). \quad (6)$$

In order to be used in on – line estimation, the computation is implemented recursively to save the computation time. The computation can be arranged in such way that the results at time $t-1$ can be used in order to get estimates at time t. The equation (6) is now written in a recursive form, but first this equation is rewritten as follow

$$\hat{\theta}(t)=\left[\sum_{i=1}^{t}\varphi(i)\varphi^T(i)\right]^{-1}\left[\sum_{i=1}^{t}\varphi(i)y(i)\right]=P(t)\left[\sum_{i=1}^{t}\varphi(i)y(i)\right] \quad (7)$$

Let $\hat{\theta}(t-1)$ denotes the least squares estimates based on the measurements at time $t-1$, the definition of $P(t)$ is:

$$P(t)^{-1}=P(t-1)^{-1}+\varphi(t)\varphi^T(t) \quad (8)$$

using equation (7) and (8) gives:

$$\sum_{i=1}^{t-1}\varphi(i)y(i)=P(t-1)^{-1}\hat{\theta}(t-1) \quad (9)$$
$$=P(t)^{-1}\hat{\theta}(t-1)-\varphi(t)\varphi^T(t)\hat{\theta}(t-1)$$

Now the estimate at time t can be written as:

$$\hat{\theta}(t)=\hat{\theta}(t-1)+K(t)\varepsilon(t) \quad (10)$$

where $K(t)=P(t)\varphi(t),\ \varepsilon(t)=y(t)-\varphi^T(t)\hat{\theta}(t-1)$

$\varepsilon(t)$ can be interpreted as the prediction error (one step ahead) of $y(t)$ based on the estimate $\hat{\theta}(t-1)$. The next step is to find a recursive equation for the update of $P(t)$. By applying the matrix inversion formula:

$$(A+BCD)^{-1}=A^{-1}-A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1}$$

to the equation (8), we have

$$P(t)=P(t-1)-\frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{I+\varphi^T(t)P(t-1)\varphi(t)} \quad (11)$$

In several adaptive control systems, the parameters θ_i are not constants. In case the parameters vary slowly in time, the least – squared criterion in (2) is replaced with:

$$J(\theta,t)=\frac{1}{2}\sum_{i=1}^{t}\lambda^{t-i}(y(i)-\varphi^T(i)\theta)^2 \quad (12)$$

The parameter $\lambda\,(0<\lambda\le 1)$ is called forgetting factor: The most recent data point have λ=1, but data points that are n time unit old are weighted by λ^n.

Repeating the calculation leading the equation (11), the follow results are obtained:

$$\hat{\theta}(t)=\hat{\theta}(t-1)+K(t)\left[y(t)-\varphi^T(t)\hat{\theta}(t-1)\right] \quad (13a)$$

$$K(t)=P(t)\varphi(t)=\frac{P(t-1)\varphi(t)}{\lambda(t)+\varphi^T(t)P(t-1)\varphi(t)} \quad (13b)$$

$$P(t)=\frac{1}{\lambda(t)}\left[P(t-1)-\frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{\lambda(t)+\varphi^T(t)P(t-1)\varphi(t)}\right] \quad (13c)$$

In order to be used in on – line identification, Bierman UD factorization algorithm [1] is applied to compute the paremater estimation. This identification algorithm is written as a S-Function in order to be used with Real Time Windows Target Toolbox of Matlab.

3. APPLICATION OF GENERALIZED PREDICTIVE CONTROL TO SMA ACTUATORS SYSTEMS

The system to be controlled is described by the following Controlled Autoregressive and Integrated Moving Average (CARIMA) model

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{C(q^{-1})}{\Delta}\xi(t) \qquad (14)$$

where $y(t)$ is the system output, $u(t)$ is the control signal, $\xi(t)$ is the process disturbance, $A(q^{-1})$ and $B(q^{-1})$ are polynomials in backward shift operator q^{-1} , $\Delta = 1 - q^{-1}$ is a different operator. In slow response SMA system, the model (14) is modified with a delay d steps and explained on the signal error as follow

$$A(q^{-1})\Delta y(t) = B(q^{-1})q^{-d}\Delta u(t-1) + C(q^{-1})\xi(t) \qquad (15)$$

due to: $\Delta y(t) = (1-q^{-1})y(t) = y(t) - y(t-1)$

$\Delta u(t) = (1-q^{-1})u(t) = u(t) - u(t-1)$

The main idea of GPC is to compute the future control sequence by minimising the following cost function in such a way that the future output $y(t)$ is driven close to reference $r(t)$:

$$J(u,t) = E\left\{\sum_{j=N_1}^{N_2}\left[y(t+j) - r(t+j)\right]^2 + \lambda\sum_{j=1}^{N_u}\left[\Delta u(t+j-1)\right]^2\right\} (16)$$

where $y(t+j)$ is the j – step prediction of the system output on the data up to time t, $r(t+j)$ is the future reference signal, λ is a weight coefficient to penalize the control sequence, and $E\{.\}$ is the expectation operator which has been used to indicate the computation of control values upon the data up to time t and the stochastic disturbance model. N_1, N_2, N_u are the minimum predictive horizon, maximum predictive horizon, and control horizon respectively.

To solve this problem, we need to know the predicted output values that are computed through the following steps.

Rewrite the model (14) as follow

$$y(t) = \frac{B(q^{-1})q^{-d}}{A(q^{-1})}u(t-1) + \frac{C(q^{-1})}{A(q^{-1})\Delta}\xi(t) \qquad (17)$$

Consider the identity:

$$\frac{C(q^{-1})}{A(q^{-1})\Delta} = E_j(q^{-1}) + \frac{q^{-j}F_j(q^{-1})}{A(q^{-1})\Delta} \qquad (18)$$

The output prediction is obtained by using Diophantine equation:

$$C(q^{-1}) = E_i(q^{-1})A(q^{-1})\Delta + q^{-j}F_i(q^{-1}). \qquad (19)$$

Therefore:

$$y(t+j) = \frac{B(q^{-1})q^{-d}}{A(q^{-1})}u(t+j-1) + E_j(q^{-1})\xi(t+j) + \frac{F_j(q^{-1})}{A(q^{-1})\Delta}\xi(t) \qquad (20)$$

Substituting $\xi(t)$ from (17) to (20) gives:

$$y(t+j) = \frac{F_j(q^{-1})}{C(q^{-1})}y(t) + \frac{E_j(q^{-1})B(q^{-1})q^{-d}}{C(q^{-1})} \times \Delta u(t+j-1) + E_j(q^{-1})\xi(t+j) \qquad (21)$$

The output at time $t+j$ is calculated from the known values at time t and the future control signal

For simplicity, the C polynomial is chosen to be 1, then the equation (21) becomes:

$$y(t+j) = F_j(q^{-1})y(t) + E_j(q^{-1})B(q^{-1})q^{-d} \times \Delta u(t+j-1) + E_j(q^{-1})\xi(t+j) \qquad (22)$$

The unknown polynomial $E(q^{-1})$, $F(q^{-1})$ is found by solving the recursion form of Diophantine equation (19). Consider two instants of time i and $i+1$:

$$1 = E_i(q^{-1})A(q^{-1})\Delta + q^{-i}F_i(q^{-1}) \qquad (23)$$

$$1 = E_{i+1}(q^{-1})A(q^{-1})\Delta + q^{-(i+1)}F_{i+1}(q^{-1}) \qquad (24)$$

where $i = 1, 2, ..., N_u$. Subtracting (23) from (24) gives:

$$0 = A(q^{-1})\Delta\left[E_{i+1}(q^{-1}) - E_i(q^{-1})\right] + q^{-i}\left[q^{-1}F_{i+1}(q^{-1}) - F_i(q^{-1})\right] \qquad (25)$$

let $R = E_{i+1}(q^{-1}) - E_i(q^{-1})$ in i^{th} step, therefore:

$$R = R_1 + r_i q^{-i} \qquad (26)$$

where $r_i = e_{i+1,i}$ is the coefficient at the most negative degree of E_{i+1}. Therefore equation (25) can be rewritten as:

$$0 = A(q^{-1})\Delta R_1 + q^{-i}\left[q^{-1}F_{i+1}(q^{-1}) - F_i(q^{-1}) + r_i A(q^{-1})\Delta\right] \qquad (27)$$

To identify two sides of equation (27), we must have:

i) $R_1 = 0$ (28)

this means that all coefficient of E_i are also the coefficient of E_{i+1}, or:

$$E_{i+1}(q^{-1}) = E_i(q^{-1}) + r_i q^{-i} \quad (29)$$

ii) $F_{i+1}(q^{-1}) = q\left[F_i(q^{-1}) - r_i A(q^{-1})\Delta\right]$ (30)

where

$F_i(q^{-1}) = f_{i0} + f_{i1}q^{-1} + ...,\ A(q^{-1}) = a_0 + a_1 q^{-1} + ...$

Equations (29) and (30) can be used to compute the values of E_i, F_i, as well as r_i. The value of r_i obtained by supposing that there is no positive degree of q in F_{i+1} formula. This means $f_{i0} - r_{i0} = 0$, due to $a_0\Delta = 1$. Therefore:

$$r_{i0} = f_{i0} \quad (31)$$

Suppose that the initial values for $E(q^{-1})$ and $F(q^{-1})$ are: $E_1(q^{-1}) = 1$ and $F_1(q^{-1}) = q\left[1 - A(q^{-1})\Delta\right]$ the updated values of E_i and F_i at each instant of time can be computed by the following procedure:

$$e_{i+1,j} = e_{i,j},\ j < i$$
$$e_{i+1,j} = f_{i,0},$$
$$f_{i+1,j} = f_{i,j+1} - e_{i+1,i}a_{j+1}\Delta,\ j = 0,1,....,n$$

where $i = 1,2,...,N_{u-1}$. The predicted output is computed from equation (22) as follow:

$$y(t+j) = F_j(q^{-1})y(t) + E_j(q^{-1})B(q^{-1})\times \times\Delta u(t+j-d-1) + E_j(q^{-1})\xi(t+j) \quad (32)$$

which can be written in the form:

$$y(t+j) = \hat{y}(t+j\,|\,k) + \tilde{y}(t+j) \quad (33)$$

the first part:

$$\hat{y}(t+j\,|\,k) = F_i(q^{-1})y(t) + E_i(q^{-1})B(q^{-1})\Delta u(t+j-d-1) \quad (34)$$

consists of two terms: one depending on the future control after time t, one depending on the measured variable at time k.

the second part $\tilde{y}(t+j) = E_j(q^{-1})\xi(t+j)$ is the future noise signal. This does not depend on the control signal. Therefore, the cost function (16) is changed to a form that depends only on the error and control signal:

$$J(u,t) = \sum_{j=N_1}^{N_2}\left[\hat{y}(t+j\,|\,k) - r(t+j)\right]^2 + \lambda\sum_{j=1}^{N_u}\left[\Delta u(t+j-1)\right]^2 \quad (35)$$

where $N_1 = d+1,\ N_2 = d+n,\ and\ N_u \le i \le n$

Equation (34) can be rewritten:

$$\hat{y}(t+j\,|\,k) = F_i(q^{-1})y(t) + G_j(q^{-1})\Delta u(t+j-d-1) \quad (36)$$

where $G_j(q^{-1}) = E_i(q^{-1})B(q^{-1})$

Hence, the future outputs can be calculated in recursive form:

$$y(t+1) = G_1\Delta u(t-d) + F_1 y(t)$$
$$y(t+2) = G_2\Delta u(t-d+1) + F_2 y(t) \quad (37)$$
$$.$$
$$y(t+N) = G_N\Delta u(t-d+N-1) + F_N y(t)$$

Let $f(t+j)$ be the component of future outputs $y(t+j)$ computed from the signals which are known at time t, therefore:

$$f(t+1) = [G_1(q^{-1}) - g_{10}]\Delta u(t-d) + F_1 y(t)$$
$$f(t+2) = [G_2(q^{-1}) - q^{-1}g_{21} - g_{20}]\Delta u(t-d) + F_2 y(t) \quad (38)$$
$$...$$

where $G_i(q^{-1}) = g_{i0} + g_{i1}q^{-1} + ...$

Then the above equations can be rewritten in the vector form:

$$\hat{y} = G\tilde{u} + f \quad (39)$$

where the vectors are all $N \times 1$:

$$f = \left[\hat{y}(t+1\,|\,t), \hat{y}(t+2\,|\,t), ..., \hat{y}(t+N_2\,|\,t)\right]^T$$
$$\tilde{u} = \left[\Delta u(t-d), \Delta u(t-d+1), ..., \Delta u(t-d+N_u-1)\right]^T$$
$$\hat{y} = \left[\hat{y}(t+1), \hat{y}(t+2), ..., \hat{y}(t+N_2)\right]^T$$

the matrix G is lower-triangular dimension $N \times N$:

$$G = \begin{bmatrix} g_0 & 0 & ... & 0 \\ g_1 & g_0 & ... & 0 \\ . & . & . & . \\ g_{N_u-1} & g_{N_u-2} & ... & g_0 \\ . & . & . & . \\ g_{N_2-1} & g_{N_2-2} & ... & g_{N_2-N_u} \end{bmatrix}$$

Furthermore, the cost function in (36) can be expressed as:

$$J = (\hat{y} - r)^T(\hat{y} - r) + \lambda\tilde{u}^T\tilde{u} \quad (40)$$

Minimize this cost function without constraints on the future controls by using $gradient(J) = 0$, we have:

$$\tilde{u} = (G^T G + \lambda I)^{-1}G^T(r - f) \quad (41)$$

where $r = \left[r(t+1), r(t+2), ..., r(t+N_2)\right]^T$ is the reference signal

In summary, the GPC algorithm is

implemented through the following steps:

- *Compute parameters for model (14)*
- *Solve Diophantine equation (19) to obtain polynomials* E_i *and* F_i*, then compute* $G_i = E_i B$
- *Determine the future output* $y(t+j)$ *by equation (33)*
- *Find control signal vector* $\tilde{u}$ *from equation (41)*
- *Let* $u_t = u_{t-1} - \Delta u_t$ *. At* $k = k+1$ *, repeat the first step.*

In this work, the program implemented GPC algorithm is written as a block S-Function in order to used with Real Time Windows Target.

4. EXPERIMENTAL RESULTS

Fig.1 shows the experimental apparatus for SMA positioning system. In this experimental setup, a small tensile SMA wire is used with some main specifications: heat current: ca.2V/0.85A, gen. force: 8N, displacement: ca.5mm. The displacement is measured by a high precision potentiometer. This system is controlled in real-time by using Advantech PCI-1711 Card with Real time Windows Target Toolbox of Matlab.

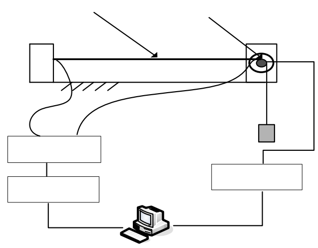

Fig. 1 Experimental apparatus

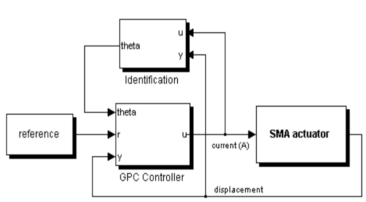

Fig.2 Schemetic diagram of predictive controller applied to SMA actuator system

GPC scheme applied to SMA actuators is shown in Fig.2. There are three inputs to GPC controller: *theta* is the estimated parameters of model (14) computed by the algorithm in section 2, *r* is the reference displacement, and *y* is the output displacement measured from SMA actuators. The input current applied to SMA actuator system is obtained from the GPC controller. The GPC controller computes a set of future control signals at each sampling time, but only the first element of control signal is applied to the system input.

The performance of this control system is experimentally investigated for different reference inputs and various parameters of the GPC controller. Fig.3 shows the performance of the control system to a pulse reference input. The control signal computed from GPC algorithm is shown in Fig.4. The parameters of GPC controller used in these experiments are tuned to produce the good response:

$N_1 = 4,\ N_2 = 20,\ N_u = 7,\ n_a = 5,\ n_b = 4, \lambda = 0.25$.

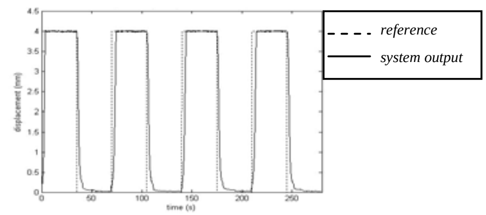

Fig. 3 System response to pulse reference input

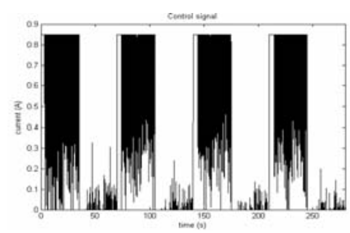

Fig.4 Control signal

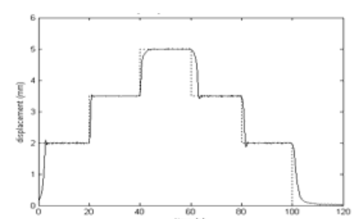

Fig. 5 Step response of GPC Controller

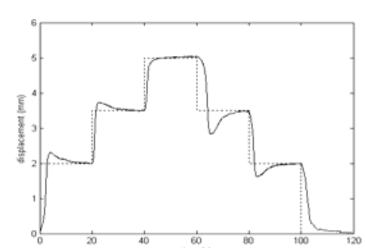

Fig. 6 Step response of PID Controller

The system response to step reference input is shown in Fig.5 and compared with conventional PID control shown in Fig.6.

From these figures, it can be observed that the GPC controller achieves better tracking response than PID controller despite the expense of more computation in implementation.

Fig.7 and Fig.8 depict the performace of control system to sine reference input with difference frequency

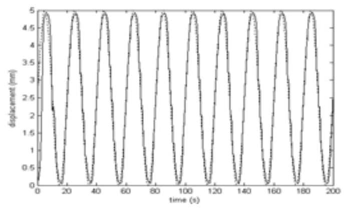

Fig. 7 System response to sine reference input (cycle: 20s)

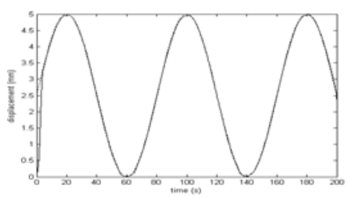

Fig. 8 System response to sine reference input (cycle: 80s)

The hysteresis curve drawn from input – output relation is shown in Fig.9. This result is obtained from the second cycle of sine reference input shown in Fig.8

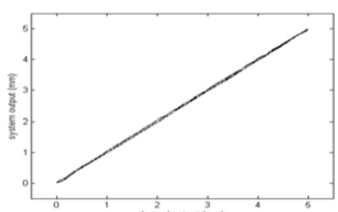

Fig. 9 Input – Output relation

Since SMA actuators are slow response systems, therefore the experimental displacement trajectory to low frequency reference input mostly overlaps the desired displacement. Fig. 9 shows the linear relationship between the desired displacement and the system output. Consequently, the GPC algorithm proves to be effective for hysteresis compensation and position control of SMA actuators.

5. CONCLUSION

In this paper, an adaptive generalized predictive controller was developed to control the position of SMA actuators. The implememtation of recursive parameter estimation and the GPC algorithm is successfully applied in real time control for SMA system. The experimental evaluation showed that the GPC controller could achieve good tracking to different reference input signals and therefore could compensate the hysteresis phenomenon of SMA actuators. The proposed controller was compared to the PID controller and was known to have better performance. This control method is not only useful applied for SMA actuators but also for other industrial processes.

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