APPLICATION OF ROBUST CONTROL TO MOBILE MANIPULATOR

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ABSTRACT

In this paper a robust control is applied to a two-wheeled mobile manipulator (WMM) to follow a smooth curved trajectory and pick an object in the 3D workplace. The dynamic motion equation of the mobile manipulator is derived taking into account parametric uncertainties, external disturbances, and the dynamic interactions between the mobile platform and the manipulator; then, a robust controller is derived based on the bounded conditions of the parameters of the total system. In addition, an USB camera is used to detect the object's coordinates in 3D space, and a practical method of camera calibration is introduced for detecting the relative position between the object and the camera which is needed for the manipulator to pick it. Also, a combined system which composed of a computer and a multi-dropped PIC-based controller is developed using USB-CAN communication to meet the performance of demand of the whole system. What's more, the simulation results are included to illustrate the performance of the robust control strategy and the camera calibration method as well.

1. INTRODUCTION

The design of intelligent, autonomous machines to perform tasks that are dull, repetitive, hazardous, or that require skill, strength, or dexterity beyond the capability of humans is the ultimate goal of robotics research. Examples of such tasks include manufacturing, excavation, construction, undersea, space, and planetary exploration, toxic waste cleanup, and robotic assisted surgery. Robotics research is highly interdisciplinary requiring the integration of control theory with mechanics, electronics, artificial intelligence, and sensor technology.

A mobile manipulator is of a manipulator mounted on a moving platform. Such the combined system has become an attraction of the researchers throughout the world. These systems, in one sense, considered to be as human body, so they can be applicable in many practical fields from industrial automation, public services to home entertainment.

In literature, a two-wheeled mobile robot has been much attracted attention because of its usefulness in many applications that need the mobility. Fierro, 1995, developed a combined kinematics and torque control law using backstepping approach and its asymptotic stability is guaranteed by Lyapunov theory which can be applied to the three basic nonholonomic navigations: trajectory tracking, path following and point stabilization [2]. Dong Kyoung Chwa et al., 2002, proposed a sliding mode controller for trajectory tracking of nonholonomic wheeled mobile robots presented in two-dimensional polar coordinates in the presence of the external disturbances [5]; T. Fukao, 2000, proposed the integration of a kinematic adaptive controller and a torque controller for the dynamic model of a nonholonomic mobile robot [4].

On the other hand, many of the fundamental theory problems in motion control of robot manipulators were solved. At the early stage, the major position control technique is known to be the computed torque control, or inverse dynamic control, which decouples each joint of the robot and linearizes it based on the estimated robot dynamic models; therefore, the performance of position control is mainly dependent upon the accurate estimations of robot dynamics. Spong and Vidyasaga [8] (1989) designed a controller based on the computed torque control for manipulators. The idea is to exactly compensate all of the coupling nonlinearities in the Lagrangian dynamics in the first stage so that the second stage compensator can be designed based on linear and decoupling plant. Moreover, a number of techniques

may be used in the second stage, such as, the method of stable factorization was applied to the robust feedback linearization problem [9] (1985). Corless and Leitmann [10] (1981) proposed a theory based on Lyapunov's second method to guaranty stability of uncertain system that can apply to the manipulators.

In this paper, a robust control based on the work of [11] was applied to two-wheeled mobile manipulator taking into account parameter uncertainties and external disturbances to pick and place an object in 3D space. To design the tracking controller, posture errors of the mobile platform and of joint space are defined; and the controllers are extracted from the bounded condition of the parameter and disturbance. In addition, a scheme of 3D position measurement using USB camera is introduced. Also, the simulation result shows the effectiveness of the designed controller.

2. DYNAMIC MODEL OF THE WMM

First, consider a two-wheeled mobile platform which can move forward, and spin about its geometric center, as shown in Fig. 1. The $2b$ is the length of the axis between the wheels of the mobile platform and r is the radius of the wheels. $\{OXY\}$ is the stationary coordinates system, or world coordinates system; $\{P_0 xy\}$ is the coordinates system fixed to the mobile robot, and P_0 is placed in the middle of the driving wheel axis; $P_c(x_c, y_c)$ is the center of mass of the mobile platform and placed in the x-axis at a distance d of P_0 ;and a is the length of the mobile platform in the direction perpendicular to the driving wheel axis. The balance of the mobile platform is maintained by a small castor whose effect we shall ignore.

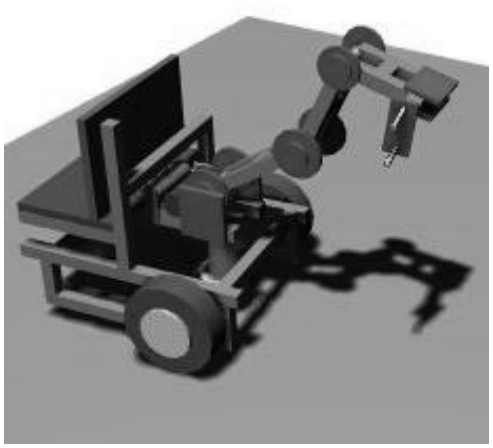

Fig. 1 Model of the mobile manipulator

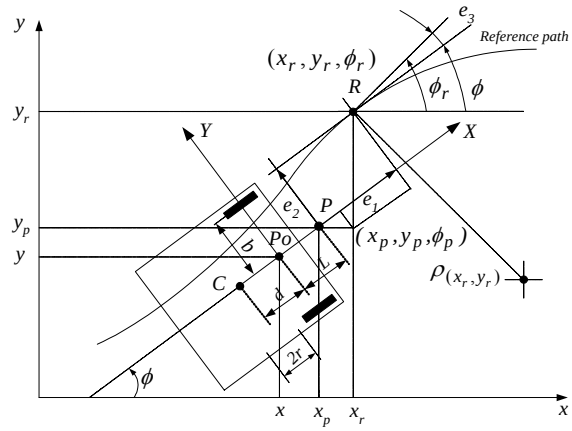

Fig. 2 Mobile platform configuration

Second, the manipulator used in this paper is of an articulated-type manipulator with two planar links in an elbow-like configuration: three rotational joints for three degrees of freedom. They are controlled by dedicated DC motors. Each joint is referred as the waist, shoulder and arm respectively. Also, the manipulator has a 3-dof end-effector function as roll, pitch and yaw; and a parallel gripper attached to the yaw-joint, which has a dedicated DC motor to control grasping objects.

The length and the center of mass of each link are presented as (L_{b1}, Z_{b1}) , (L_{b2}, Z_{b2}) , (L_{b3}, Z_{b3}) , (L_{b4}, Z_{b4}) , (L_{b5}, Z_{b5}) , respectively. The geometric model and the coordinate composed for each link is shown in Fig. 3. The model of the mobile manipulator is shown in Fig. 3.

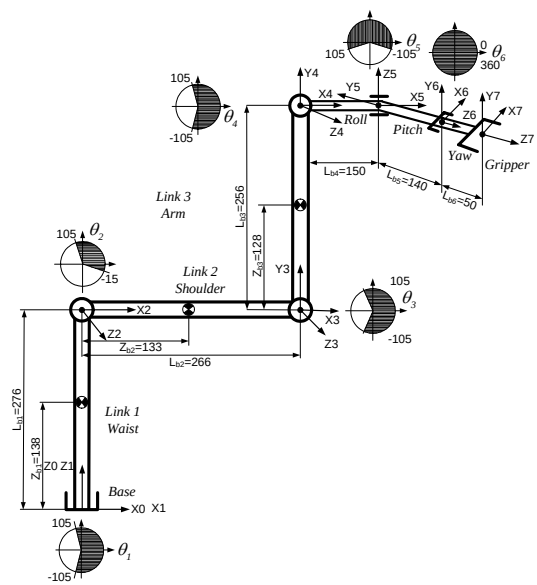

Fig. 3 Geometry of 6-dof manipulator

The dynamics of the mobile manipulator subject to kinematics constraints is given in the following form [11]:

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} \ddot{q}_v \\ \ddot{q}_a \end{pmatrix}+\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}\begin{pmatrix} \dot{q}_v \\ \dot{q}_a \end{pmatrix}+\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}+\begin{pmatrix} A_v^T(q_v)\lambda \\ 0 \end{pmatrix}+\begin{pmatrix} \tau_{d1} \\ \tau_{d2} \end{pmatrix}=\begin{pmatrix} E_v\tau_v \\ \tau_a \end{pmatrix} \quad (1)$$

The system constraint Eq. (1) can be simplified to the nonholonomic constraint of the mobile platform only as follows:

$$A_v(q_v)\dot{q}_v = 0 \quad (2)$$

where $\tau_v \in R^{m-r}$ represents the actuated torque vector of the constrained coordinates; $E_v \in R^{mx(m-r)}$, the input transformation matrix; $\tau_b \in R^n$, the actuating torque vector of the free coordinates; τ_{d1} and τ_{d2}, disturbance torques.

According to the standard matrix theory, there exists a full rank matrix $S_v(q_v) \in R^{mx(m-r)}$ made up by a set of smooth and linearly independent vector spanning the null space of A_v, that is, $S^T(q_v)A_v^T(q_v) = 0$. From Eq. (2), we can find a velocity input vector $\eta(t) \in R^{m-n}$ such that, for all t,

$$\dot{q}_v = S(q_v)\eta \quad (3)$$

The Eq. (3) is called the *steering system*, and η is known as a velocity input to steer the state vector q in state space. Furthermore, $S(q_v)$ is bounded by $S(q_v) \le \zeta_s$, ζ_s is a positive number.

2.1 Tracking Controller for the Mobile Platform

The tracking errors with respect to a frame fixed on the mobile platform are given as follows [2]:

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (4)$$

The Lyapunov function is chosen as

$$V_0 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1-\cos e_3}{k_2} \quad (5)$$

The derivative of V_0 can be derived as

$$\dot{V}_0 = e_1(-v + v_r\cos e_3) + \frac{\sin e_3}{k_2}(\omega_r - \omega + k_2 e_2 v_r) \quad (6)$$

The velocity control law η_d achieves stable tracking of the mobile platform for the kinematic model as

$$\eta_d = \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} = \begin{bmatrix} v_r\cos e_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3 \sin e_3 \end{bmatrix} \quad (7)$$

where k_1, k_2 and k_3 are positive values.
The Eq. (7) becomes

$$\dot{V}_0 = -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 \le 0 \quad (8)$$

2.2 Lyapunov function for the mobile platform

Consider the first m-equation of Eq. (1) as follows:

$$M_{11}\ddot{q}_v + M_{12}\ddot{q}_a + C_{11}\dot{q}_v + C_{12}\dot{q}_a + F_1 + A_v^T\lambda + \tau_{d1} = E_v\tau_v \quad (9)$$

Multiplying both sides by S^T and using Eq. (3) to eliminate the constraint force term λ, it yields

$$\overline{M}_{11}\dot{\eta} + \overline{C}_{11}\eta + f_1 + \overline{\tau}_{d1} = \overline{\tau}_v \quad (10)$$

Here $\overline{M}_{11} = S^T M_{11} S$, $\overline{C}_{11} = S^T C_{11} S + S^T M_{11}\dot{S}$, $f_1 = S^T(M_{12}\ddot{q}_a + C_{12}\dot{q}_a + F_1)$, $\overline{\tau}_{d_1} = S^T\tau_{d1}$, $\overline{\tau}_v = S^T E_v \tau_v$, and $S^T E_v = \begin{bmatrix} 1/r & 1/r \\ b/r & b/r \end{bmatrix}$

It can be seen that f_1, which consists of the gravitational and friction force vector F_1 and the dynamics interaction with the manipulator $(M_{12}\ddot{q}_a + C_{12}\dot{q}_a)$, and the disturbances on the mobile platform (terrain disturbance force) needs to be compensated online.

Property 1: $\dot{\overline{M}}_{11} - 2\overline{C}_{11}$ is skew-symmetric

Property 2: $\|\overline{M}_{11}\| = \|S^T M_{11} S\| \le \overline{M}_{11b}$ and $\|M_{12}\| \le M_{12b}$

Property 3: $\|\overline{C}_{11}\| = \|S^T C_{11} S\| \le \overline{C}_{11b}\|\dot{q}\|$ and $\|C_{12}\| \le C_{12b}\|\dot{q}\|$

Assumption 1: Disturbance on the mobile platform is bounded, that is, $\|\tau_{d1}\| \le \tau_{N1}$, with τ_{N1} is a positive constant.

Assumption 2: The friction and gravity on the mobile platform are bounded by $\|F_1(q_v, \dot{q}_v)\| \le \xi_0 + \xi_1\|\dot{q}\|$, ξ_0 and ξ_1 representing some positive constants.

The *velocity tracking error* is defined as

$$\tilde{\eta} = \eta - \eta_d \quad (11)$$

then, the mobile platform dynamics in terms of velocity tracking error is derived as

$$\overline{M}_{11}\dot{\tilde{\eta}} + \overline{C}_{11}\tilde{\eta} + \overline{M}_{11}\dot{\eta}_d + \overline{C}_{11}\eta_d + f_1 + \bar{\tau}_{d1} = \bar{\tau}_v \quad (12)$$

Let us consider the following Lyapunov function

$$V_1 = \frac{1}{2}\tilde{\eta}^T \overline{M}_{11}\tilde{\eta} \quad (13)$$

and the derivative of V_1 can be derived as follows:

$$\dot{V}_1 = \tilde{\eta}^T(\bar{\tau}_v - \overline{M}_{11}\dot{\eta}_d - \overline{C}_{11}\eta_d - f_1 - \bar{\tau}_{d1}) \quad (14)$$

2.3 Lyapunov function of the manipulator

Consider the last n-equations of Eq. (1),

$$M_{22}\ddot{q}_a + C_{22}\dot{q}_a + (M_{21}\ddot{q}_v + C_{21}\dot{q}_v + F_2) + \tau_{d2} = \tau_a \quad (15)$$

Equation (15) represents the dynamic equation of the manipulator. In this equation, the unknown terms need to be compensated are the gravitational and friction force F_2, the dynamic interaction term $(M_{21}\ddot{q}_v + C_{21}\dot{q}_v)$, and the disturbances on the manipulator.

Property 4: $\dot{\overline{M}}_{22} - 2\overline{C}_{22}$ is skew-symmetric

Property 5: $\|M_{21}\| \le M_{21b}$ and $\|M_{22}\| \le M_{22b}$

Property 6: $\|C_{22}\| \le C_{22b}\|\dot{q}\|$ and $\|C_{21}\| \le C_{21b}\|\dot{q}\|$

Assumption 3: Disturbance on the manipulator is bounded, that is, $\|\tau_{d2}\| \le \tau_{N2}$, with τ_{N2} is a positive constant.

Assumption 4: Friction and gravity in the manipulator dynamics Eq. (15) are bounded by $\|F_2(q,\dot{q})\| \le \xi_2 + \xi_3\|\dot{q}\|$, ξ_2 and ξ_3 representing some positive constants.

Let us define *the joint tracking errors* and taking its derivative as

$$\begin{aligned} \tilde{q}_a &= q_{ad} - q_a \\ \dot{\tilde{q}}_a &= \dot{q}_{ad} - \dot{q}_a \end{aligned} \quad (16)$$

Also, the *filter tracking error* and its derivative,

$$\begin{aligned} r_a &= \dot{\tilde{q}}_a + k\tilde{q}_a, k = k^T > 0 \\ \dot{r}_a &= \ddot{\tilde{q}}_a + k\dot{\tilde{q}}_a = \ddot{q}_{ad} - \ddot{q}_a + k(r_a - k\tilde{q}_a) \end{aligned} \quad (17)$$

The manipulator dynamics equation can be formulated in terms of filtered tracking error as follows:

$$-M_{22}\dot{r}_a + (M_{22}k - C_{22})(r_a - k\tilde{q}_a) + f_2 + \tau_{d2} = \tau_a \quad (18)$$

where $f_2 = M_{22}\ddot{q}_{ad} + C_{22}\dot{q}_{ad} + (M_{21}\ddot{q}_v + C_{21}\dot{q}_v + F_2)$

The Lyapunov function for the manipulator is defined as

$$V_2 = \frac{1}{2}r_a^T M_{22} r_a \quad (19)$$

the time derivative of V_2 can be derived as follows

$$\begin{aligned} \dot{V}_2 &= r_a^T M_{22}\dot{r}_a + \frac{1}{2}r_a^T \dot{M}_{22} r_a \\ &= r_a^T[-\tau_a - (M_{22}k - C_{22})k\tilde{q}_a + M_{22}kr_a + f_2 + \tau_{d2}] \end{aligned} \quad (20)$$

2.4 Lyapunov function of the mobile manipulator

The Lyapunov function for the overall system, the mobile platform and the manipulator, can be defined and rearrange as follows:

$$\begin{aligned} V &= V_0 + \frac{1}{2}\begin{pmatrix} S\tilde{\eta} \\ -r_a \end{pmatrix}^T \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{pmatrix} \begin{pmatrix} S\tilde{\eta} \\ -r_a \end{pmatrix} \\ &= V_0 + \frac{1}{2}(S\tilde{\eta})^T M_{11}(S\tilde{\eta}) - \frac{1}{2}r_a^T M_{12}^T(S\tilde{\eta}) - \frac{1}{2}(S\tilde{\eta})^T M_{12} r_a + \frac{1}{2}r_a^T M_{22} r_a \\ &= V_0 + \frac{1}{2}\tilde{\eta}^T(S^T M_{11} S)\tilde{\eta} - r_a^T M_{12}^T(S\tilde{\eta}) + \frac{1}{2}r_a^T M_{22} r_a \\ &= V_0 + \frac{1}{2}\tilde{\eta}^T \overline{M}_{11}\tilde{\eta} - r_a^T M_{12}^T(S\eta) + \frac{1}{2}r_a^T M_{22} r_a \\ &= V_0 + V_1 + V_2 - r_a^T M_{12}^T(S\tilde{\eta}) \end{aligned} \quad (21)$$

Taking the time derivative of V yields

$$\dot{V} = \dot{V}_0 + \dot{V}_1 + \dot{V}_2 - \frac{d}{dt}\left\{r_a^T M_{21}(S\tilde{\eta})\right\} \quad (22)$$

Substituting (14), (20) into (22) yields

$$\begin{aligned} \dot{V} &= \dot{V}_0 + \tilde{\eta}^T(\bar{\tau}_v - \overline{M}_{11}\dot{\eta}_d - \overline{C}_{11}\eta_d - f_1 - \bar{\tau}_{d1}) \\ &+ r_a^T[-\tau_a - (M_{22}k - C_{22})k\tilde{q}_a + M_{22}kr_a + f_2 + \tau_{d2}] - \frac{d}{dt}\left\{r_a^T M_{21}(S\tilde{\eta})\right\} \end{aligned} \quad (23)$$

On the other hand, f_1 can be rewritten in terms of error tracking filter r_a as follows:

$$\begin{aligned} f_1 &= S^T(M_{12}\ddot{q}_a + C_{12}\dot{q}_a + F_1) \\ &= S^T\{M_{12}[\ddot{q}_{ad} - \dot{r}_a + k(r_a - k\tilde{q}_a)] + C_{12}[\dot{q}_{ad} - (r_a - k\tilde{q}_a)] + F_1\} \\ &= -S^T[M_{12}\dot{r}_a + (C_{12} - M_{12}k)(r_a - k\tilde{q}_a)] + S^T(M_{12}\ddot{q}_{ad} + C_{12}\dot{q}_{ad} + F_1) \\ &= -S^T[M_{12}\dot{r}_a + (C_{12} - M_{12}k)(r_a - k\tilde{q}_a)] + \bar{f}_1 \end{aligned} \quad (24)$$

with $\bar{f}_1 = S^T(M_{12}\ddot{q}_{ad} + C_{12}\dot{q}_{ad} + F_1)$

Similarly, f_2, in terms of velocity error $\tilde{\eta}$ as follows:

$$\begin{aligned} f_2 &= M_{21}\ddot{q}_v + C_{21}\dot{q}_v + (M_{22}\ddot{q}_{ad} + C_{22}\dot{q}_{ad} + F_2) \\ &= M_{21}(\dot{S}\eta + S\dot{\eta}) + C_{21}S\eta + (M_{22}\ddot{q}_{ad} + C_{22}\dot{q}_{ad} + F_2) \\ &= (M_{21}S)\dot{\eta} + (M_{21}\dot{S} + C_{21}S)\eta + \bar{f}_2 \\ &= (M_{21}S)(\dot{\tilde{\eta}} + \dot{\eta}_d) + (M_{21}\dot{S} + C_{21}S)(\tilde{\eta} + \eta_d) + \bar{f}_2 \end{aligned} \quad (25)$$

with $\bar{f}_2 = (M_{22}\ddot{q}_{ad} + C_{22}\dot{q}_{ad} + F_2)$

Substituting (24) and (25) into (23), we have

$$\dot{V} = V_0 + \tilde{\eta}^T \{\bar{\tau}_v - \psi_1\} - \tilde{\eta}^T \bar{\tau}_{d1} + r_a^T \{-\tau_a + \psi_2\} + r_a^T \tau_{d2} \quad (26)$$

where

$$\psi_1 = \overline{M}_{11}\dot{\eta}_d + \overline{C}_{11}\eta_d + \bar{f}_1 + S^T \{C_{12}k\tilde{q}_a + M_{12}k(r_a - k\tilde{q}_a)\}$$
$$\psi_2 = M_{22}kr_a + (C_{22} - M_{22}k)k\tilde{q}_a + \bar{f}_2 + M_{21}S\dot{\eta}_d + M_{21}\dot{S}\eta_d + C_{21}S\eta_d$$

The nonlinear terms ψ_1 and ψ_2 are need to be identical online using robust control scheme in the following section based on the work in [14].

3. ROBUST CONTROLLER DESIGN

First, consider the second term of the Eq. (26) and using **Properties** (1)-(3) and **Assumptions** (1)-(2):

$$\begin{aligned}
&\tilde{\eta}^T \left\{\bar{\tau}_v - \overline{M}_{11}\dot{\eta}_d - \overline{C}_{11}\eta_d - \bar{f}_1 - S^T[C_{12}k\tilde{q}_a + M_{12}k(r_a - k\tilde{q}_a)]\right\} - \tilde{\eta}^T \bar{\tau}_{d1} \\
&= \tilde{\eta}^T \bar{\tau}_v + \tilde{\eta}^T \left\{\begin{array}{l} -\overline{M}_{11}\dot{\eta}_d - S^T M_{12}[\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)] - \overline{C}_{11}\eta_d - \\ S^T C_{12}[\dot{q}_{ad} + k\tilde{q}_a] - S^T F_1 - S^T \tau_{d1} \end{array}\right\} \\
&\le \tilde{\eta}^T \bar{\tau}_v + \tilde{\eta}^T \left\{\begin{array}{l} \|\overline{M}_{11}\|\|\dot{\eta}_d\| + S^T \|M_{12}\|\|\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)\| + \\ \|\overline{C}_{11}\|\|\eta_d\| + S^T \|C_{12}\|\|\dot{q}_{ad} + k\tilde{q}_a\| + S^T \|F_1\| + S^T \|\tau_{d1}\| \end{array}\right\} \\
&\le \tilde{\eta}^T \bar{\tau}_v + \|\tilde{\eta}\| \left\{\begin{array}{l} \overline{M}_{11b}\|\dot{\eta}_d\| + \varsigma_s M_{12b}\|\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)\| + \overline{C}_{11b}\|\eta_d\|\|\dot{q}\| + \\ \varsigma_s C_{12b}\|\dot{q}_{ad} + k\tilde{q}_a\|\|\dot{q}\| + \varsigma_s(\xi_0 + \xi_1\|\dot{q}\| + \tau_{N1}) \end{array}\right\} \\
&= \tilde{\eta}^T \bar{\tau}_v + \|\tilde{\eta}\| \Delta_1^T \varphi_1
\end{aligned} \quad (27)$$

where the unknown vector Δ_1^T and the robust damping vector φ_1 are defined in the following:

$$\begin{aligned}
\Delta_1^T &= (\overline{M}_{11b}, \varsigma_s M_{12b}, \overline{C}_{11b}, \varsigma_s C_{12b}, \xi_1\varsigma_s, (\xi_0 + \tau_{N1})\varsigma_s) \\
\varphi_1^T &= (\|\dot{\eta}_d\|, \|\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)\|, \|\eta_d\|\|\dot{q}\|, \|\dot{q}_{ad} + k\tilde{q}_a\|\|\dot{q}\|, \|\dot{q}\|, 1)
\end{aligned} \quad (28)$$

Second, consider the third term of the Eq. (26) and using **Properties** (4)-(6) and **Assumptions** (3)-(4):

$$\begin{aligned}
&r_a^T \begin{bmatrix} -\tau_a + M_{22}kr_a + (C_{22} - M_{22}k)k\tilde{q}_a + \bar{f}_2 + M_{21}S\dot{\eta}_d + \\ M_{21}\dot{S}\eta_d + C_{21}S\eta_d \end{bmatrix} + r_a^T \tau_{d2} \\
&\le -r_a^T \tau_a + r_a^T \left\{\begin{array}{l} M_{22}[\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)] + M_{21}(S\dot{\eta}_d + \dot{S}\eta_d) + \\ C_{22}(\dot{q}_{ad} + k\tilde{q}_a) + C_{21}S\eta_d + F_2 + \tau_{d2} \end{array}\right\} \\
&\le -r_a^T \tau_a + r_a^T \left\{\begin{array}{l} \|M_{22}\|\|\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)\| + \|M_{21}\|\|S\dot{\eta}_d + \dot{S}\eta_d\| + \\ \|C_{22}\|\|\dot{q}_{ad} + k\tilde{q}_a\| + \|C_{21}\|S\|\eta_d\| + \|F_2\| + \|\tau_{d2}\| \end{array}\right\} \\
&\le -r_a^T \tau_a + \|r_a\| \left\{\begin{array}{l} M_{22b}\|\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)\| + M_{21b}\|S\dot{\eta}_d + \dot{S}\eta_d\| + \\ C_{22b}\|\dot{q}_{ad} + k\tilde{q}_a\|\|\dot{q}\| + C_{21b}\varsigma_s\|\eta_d\|\|\dot{q}\| + \xi_2 + \xi_3\|\dot{q}\| + \tau_{N2} \end{array}\right\} \\
&= -r_a^T \tau_a + \|r_a\| \Delta_2^T \varphi_2
\end{aligned} \quad (29)$$

where the unknown vector Δ_2^T and the RDC vector φ_2 are defined as follows:

$$\Delta_2^T = (M_{22b}, M_{12b}, C_{22b}, C_{12b}\varsigma_s, \xi_3, \xi_2 + \tau_{N2})$$
$$\varphi_2^T = (\|\ddot{q}_{ad} + k(r_a - k\tilde{q}_a)\|, \|S\dot{\eta}_d + \dot{S}\eta_d\|, \|\dot{q}_{ad} + k\tilde{q}_a\|\|\dot{q}\|, \|\eta_d\|\|\dot{q}\|, \|\dot{q}\|, 1)$$

Let us choose the mobile platform and manipulator torque inputs as

$$\bar{\tau}_v = -k_{pv}\tilde{\eta} - k_{11}\tilde{\eta}\|\varphi_1\|^2 \quad (30)$$

$$\tau_a = k_{pa} r_a + k_{22} r_a \|\varphi_2\|^2 \quad (31)$$

where $k_{pv} \ge 0$, $k_{pa} \ge 0$, $k_{11} \ge 0$,and $k_{22} \ge 0$ are the controller gains; φ_1 and φ_2 are the robust damping control vectors, respectively. Then the tracking errors of the closed-loop system are guaranteed to be globally uniformly ultimately bounded.

Proof: substituting (30) and (31) into (26) yields

$$\begin{aligned}
\dot{V} &\le -V_0 - k_{pv}\tilde{\eta}^T\tilde{\eta} - k_1\tilde{\eta}^T\tilde{\eta}\|\varphi_1\|^2 + \tilde{\eta}^T \Delta_1^T \varphi_1 - k_{pa} r_a^T r_a - k_2 r_a^T r_a \|\varphi_2\|^2 + \|r_a\| \Delta_2^T \varphi_2 \\
&\le -V_0 - k_1\|\eta_d\|^2\|\varphi_1\|^2 + \|\eta_d\|\|\Delta_1\|\|\varphi_1\| - k_2\|r_a\|^2\|\varphi_2\|^2 + \|r_a\|\|\Delta_2\|\|\varphi_2\| \\
&= -V_0 - k_1\left(\|\eta_d\|^2\|\varphi_1\|^2 - \frac{\|\eta_d\|\|\Delta_1\|\|\varphi_1\|}{k_1}\right) - k_2\left(\|r_a\|^2\|\varphi_2\|^2 - \frac{\|r_a\|\|\Delta_2\|\|\varphi_2\|}{k_2}\right) \\
&= -V_0 - k_1\left[\left(\|\eta_d\|\|\varphi_1\| - \frac{\|\Delta_1\|}{2k_1}\right)^2 - \frac{\|\Delta_1\|^2}{4k_1^2}\right] - k_2\left[\left(\|r_a\|\|\varphi_2\| - \frac{\|\Delta_2\|}{2k_2}\right)^2 - \frac{\|\Delta_2\|^2}{4k_2^2}\right] \\
&= -V_0 - k_1\left[\left(\|\eta_d\|\|\varphi_1\| - \frac{\|\Delta_1\|}{2k_1}\right)^2\right] - k_2\left[\left(\|r_a\|\|\varphi_2\| - \frac{\|\Delta_2\|}{2k_2}\right)^2\right] + \frac{\|\Delta_1\|^2}{4k_1} + \frac{\|\Delta_2\|^2}{4k_2} \\
&< -k_{\min}\left\{\left(\|\eta_d\|\|\varphi_1\| - \frac{\|\Delta_1\|}{2k_1}\right)^2 + \left(\|r_a\|\|\varphi_2\| - \frac{\|\Delta_2\|}{2k_2}\right)^2\right\} + \frac{\|\Delta_{\max}\|^2}{2k_{\min}}
\end{aligned} \quad (32)$$

where $k_{\min} = \min\{k_1, k_2\}$ and $\|\Delta_{\max}\| = \max\{\|\Delta_1\|, \|\Delta_2\|\}$

In Eq. (32), $\|\Delta_{\max}\|$ is a bounded quantity; therefore, V decreases monotonically until the solutions reach a compact set determined by the right-hand-side of Eq. (32). The size of the residual set can be decreased by increasing $k_{\min}$. According to the standard Lyapunov theory and the extension of the LaSalle theory, this demonstrates that the control input Eqs. (30) and (31) may guarantee global uniform ultimate boundedness of all tracking errors.

4. CONTROL SYSTEM DEVELOPMENT

The control system is based on the integration of computer and PIC-based microprocessor. The computer functions as high as high level control for image processing and control algorithm and the microprocessor, as low level controller for device control. The configuration diagram of the overall control system is shown in Fig. 4. In the configuration, there are 8 Servo-CAN modules are used to control all low-level devices: waist motor, shoulder motor, arm motor, roll motor, pitch motor, and yaw motor for manipulator; and left-wheeled and right-wheeled

motors for mobile platform. The positions of the manipulator's joints feedback to the computer via ADC module on USB-CAN. The Servo-CAN module is based on PIC18F458 with current feedback for torque control shown in Fig. 7. The USB-CAN module is used to interface between high and low levels: it transforms the serial data from computer to CAN messages shown in Fig. 6. The USB-CAN interface is shown in Fig. 5.

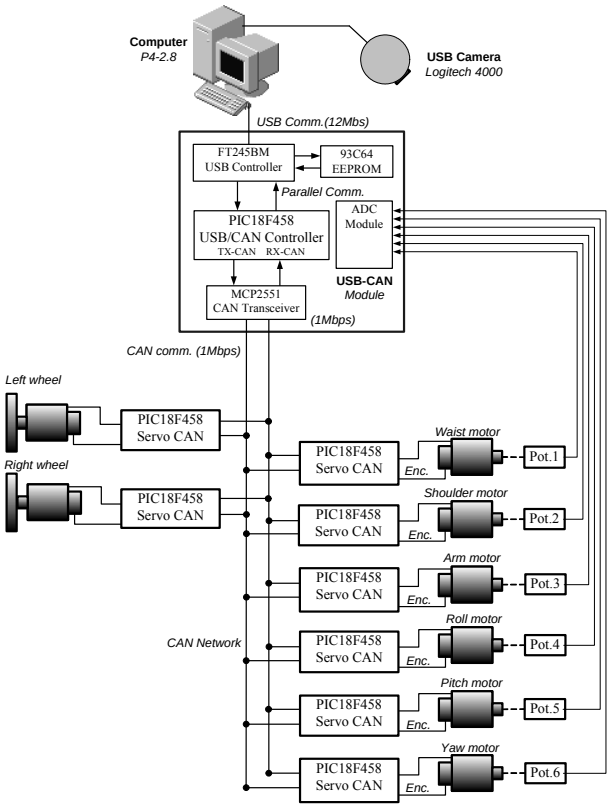

Fig. 4 Block diagram of the control system

Fig. 5 USB-CAN Interface

For the operation, USB camera Logitech 4000 is used to capture the image stream into memory with size of 320x240 at 30fps using QuickCam SDK. The image is processed using image processing library OpenCV to extract the features from the image for the object's position detection. The torque command is sent to the low level to control the mobile platform and the manipulator to pick the object. The total control processes are programmed and integrated into the interface IMR V.1 in Visual C++ shown in Fig. 8. With this interface, the mobile manipulator's parameters can be set before the operation.

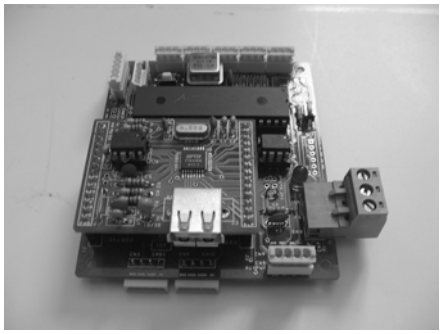

Fig. 6 USB-CAN module

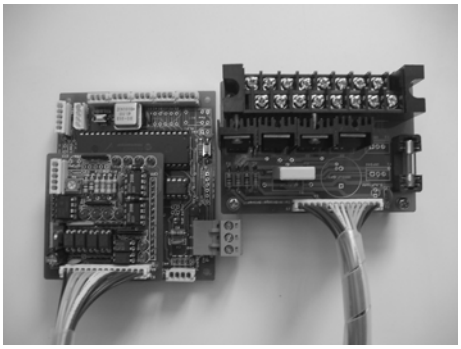

Fig. 7 Servo CAN module

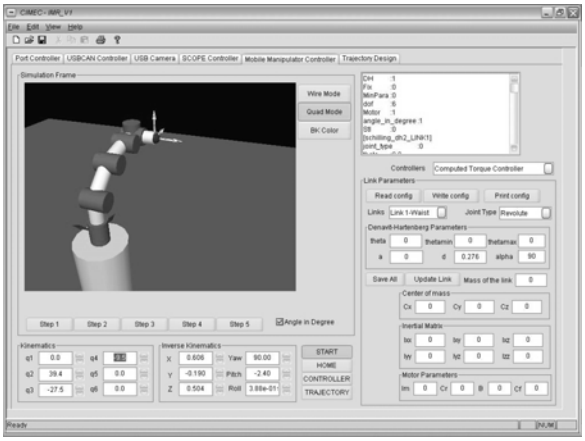

Fig. 8 Mobile manipulator interface IMR V.1

5. SIMULATION RESULTS

To verify the effectiveness of the controller, the simulations have been done with controller (7),(30) and (31) using Visual C++6 and Gnuplot 4. The reference trajectory are planned for the total system: a cubic spline reference line for the mobile platform shown in Fig. 9 with the trajectory parameters in Fig. 10 and sinusoid trajectory for joint 1 to joint 6 with the frequency of $0.8f, 0.9f, f, 1.1f, 1.2f$ and $1.3f$ ($f = 0.3125Hz$) as shown in Fig. 19, respectively. The kinematic controller constants are $k1 = 5, k_2 = 20$ and $k_3 = 10$. The robust controller gains are $k_{pv} = 40, k_{11} = 0.001, k_{pa} = 300, k_{22} = 1.2$. The mobile robot for the simulation has the following parameters: $b = 200mm$, $r = 110mm$, $m_c = 20.6kg$, $m_w = 1.2kg$. The initial posture of the mobile manipulator and the reference trajectory is $x(0) = 0.1m$, $y(0) = 0.3m$, $\phi(0) = 45^\circ$, $x_r(0) = 0.3m$, $y_r(0) = 0.5m$, $\phi_r(0) = 0$, respectively; the initial joint positions, $q_a(0) = (\pi/6, \pi/10, \pi/18, -\pi/18, -\pi/10, -\pi/6)$. Sampling time is 10ms. The disturbance is a random noise with the magnitude of 3, but it is not considered in this simulation.

Simulation results are given through Figs. 9-27. The mobile platform's tracking errors are given in Fig. 11 for full time (8s) and Fig. 12 for the initial time (2s), respectively. It can be seen that the platform errors go to zero after about 1 second. The kinematics control input, the linear and angular velocities at mobile platform center, is shown in Fig. 13; the velocities of the left and right wheel, in Fig. 14. It is shown that the linear velocity of the mobile platform is rather high in the vicinity of $2m/s$; it should be tuned for the acceptable value in the practical applications. The robust vector for the mobile platform $\varphi_1 (i = 1..6)$ in the controller Eq. (30) is given in Fig. 15, and the torque on the left and the right wheel, in Fig, 16. The mobile platform's tracking position with respect to the reference trajectory is shown in Fig. 17 for the initial time (2s) and in Fig. (18) for the full time (8s) estimated value $\hat{\mu}$ are shown in Fig. 12. The linear and angular velocities and wheel velocities of the mobile robot are given in Figs. 8 and 9, respectively. It can be seen that the tracking velocity is in the vicinity of 2m/s as desired. For the manipulator, the tracking positions of joint 1 to joint 6 are shown in Figs. 20-25. It can be seen that the overall tracking performance is acceptable, but they each are depend on the frequency of the reference trajectory. The joint's position errors and joint's torques are shown in Figs. 26 and 27, respectively.

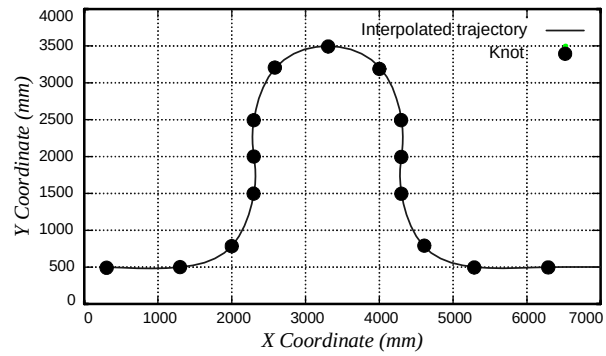

Fig. 9 Cubic spline reference trajectory for platform

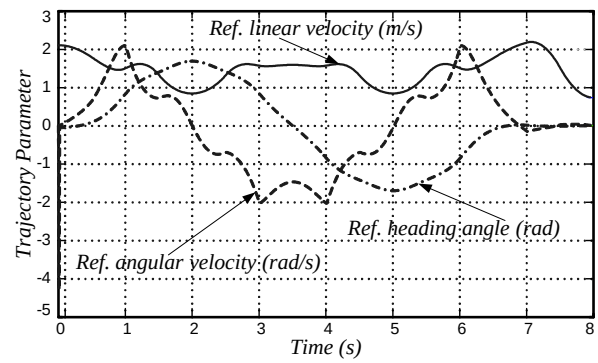

Fig. 10 Reference trajectory parameters for platform

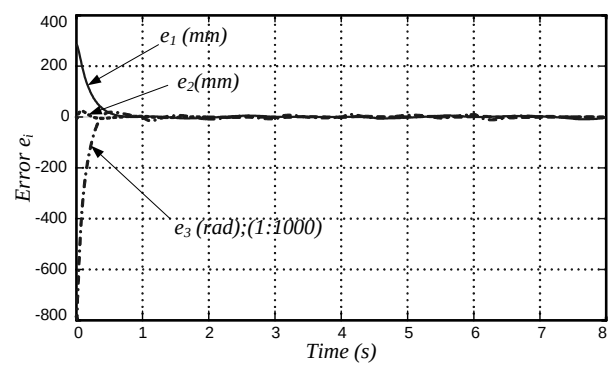

Fig. 11 Platform tracking errors for full time

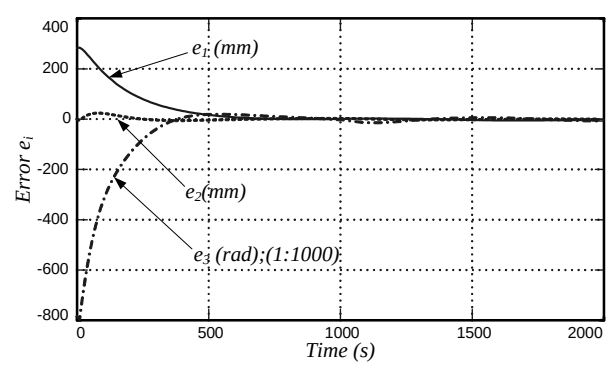

Fig. 12 Platform tracking errors for the initial time

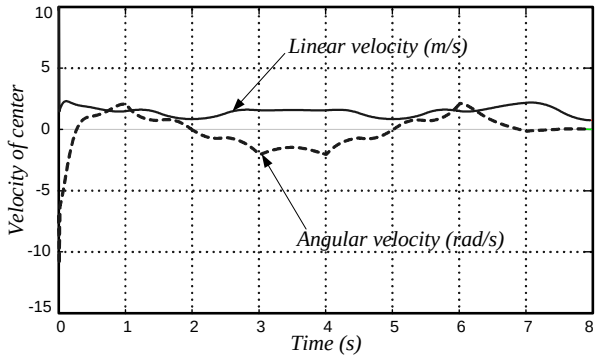

Fig. 13 Velocities of platform's center

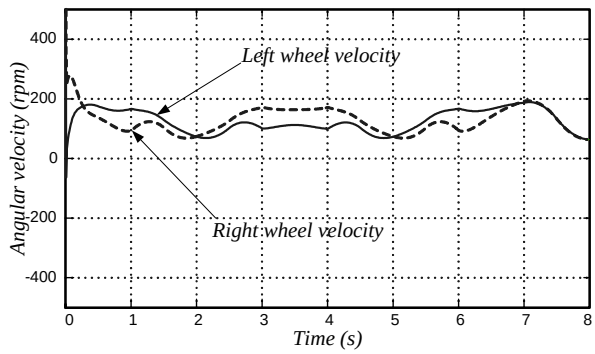

Fig. 14 Wheel velocities

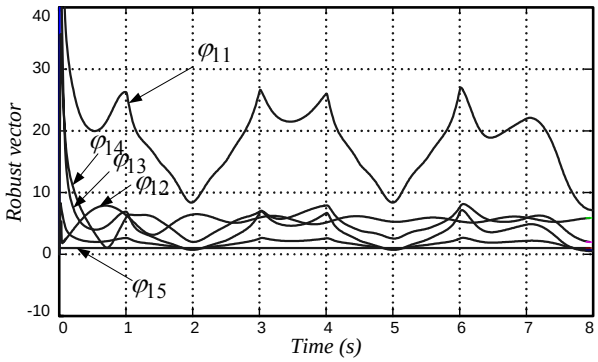

Fig. 15 Robust vector of mobile platform

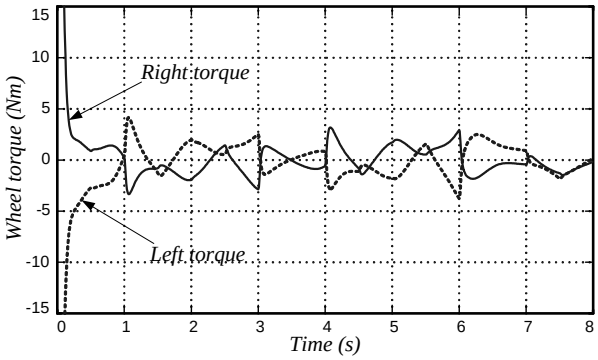

Fig. 16 Wheel's torques of mobile platform

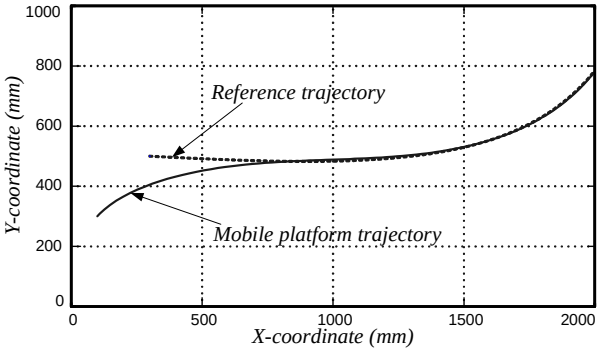

Fig. 17 Tracking trajectory of mobile platform (2s)

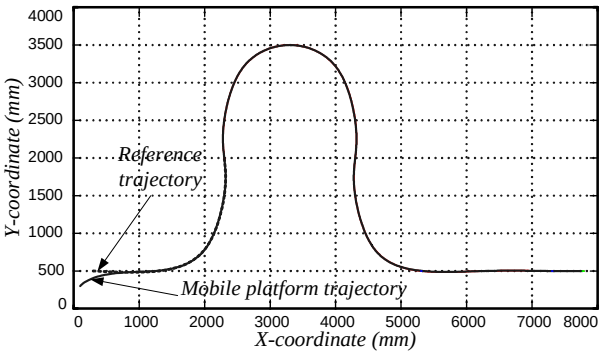

Fig. 18 Tracking trajectory of mobile platform (8s)

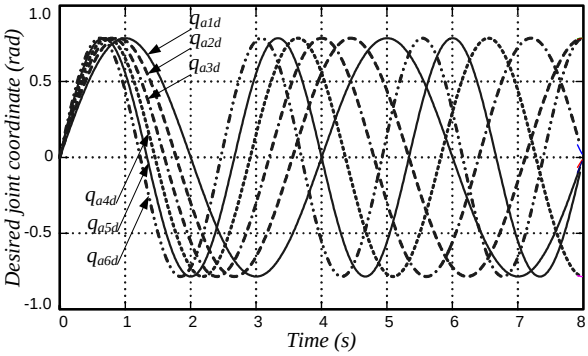

Fig. 19 Joint reference trajectory

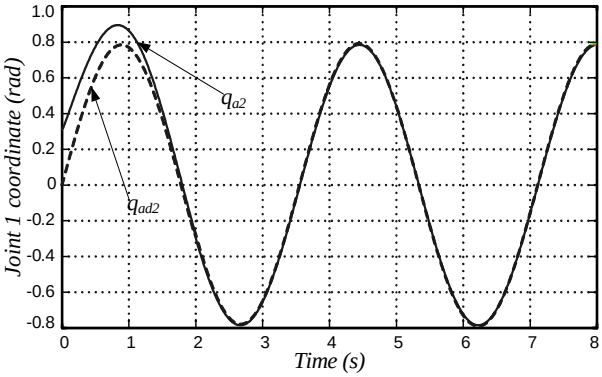

Fig. 20 Tracking position of joint 1

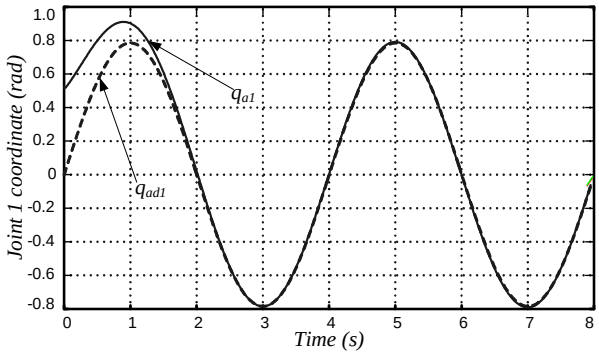

Fig. 21 Tracking position of joint 2

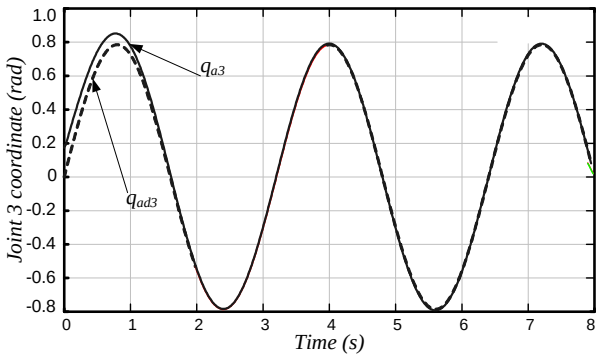

Fig. 22 Tracking position of joint 3

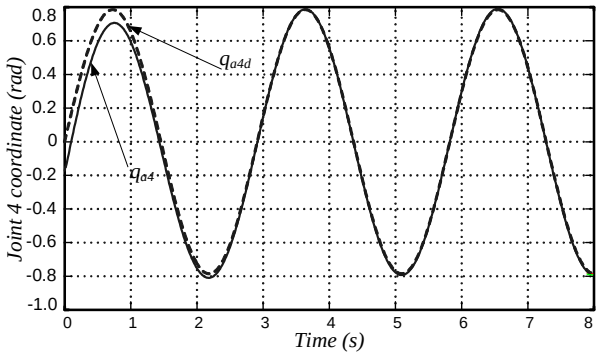

Fig. 23 Tracking position of joint 4

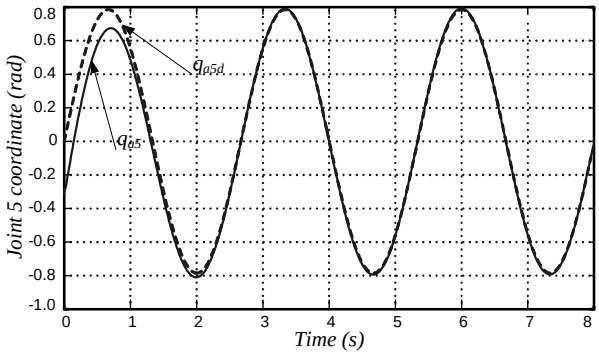

Fig. 24 Tracking position of joint 5

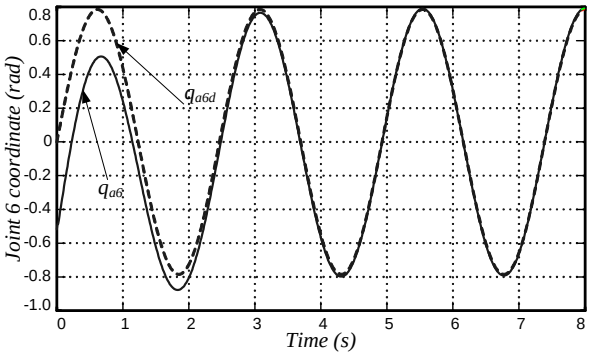

Fig. 25 Tracking position of joint 6

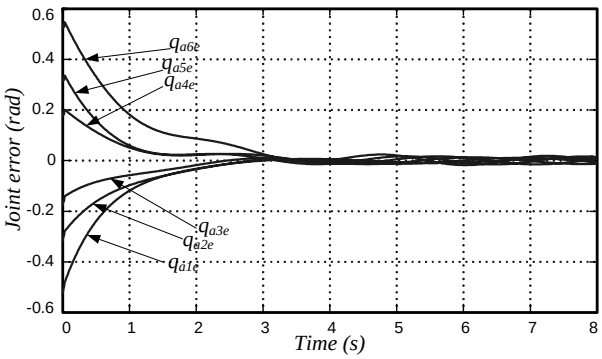

Fig. 26 Joint position error

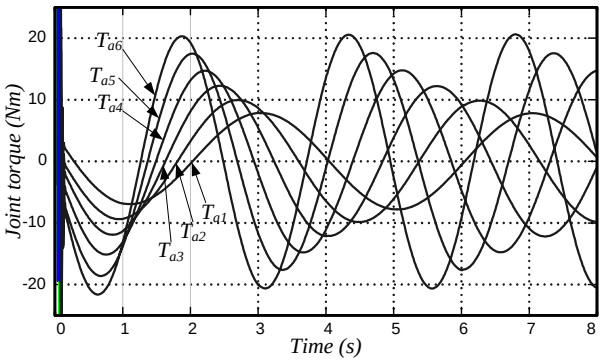

Fig. 27 Joint torque

Finally, the control system was set up, but the experimental data have not been to achieve at the time of this writing. The experimental result will be included in the future work. The mobile manipulator for the experiment is shown in Fig. 28.

Fig. 28 The experimental mobile manipulator

6. CONCLUSIONS

The robust controller was designed for the mobile manipulator to pick and place an object in 3D working space. The mobile manipulator is considered in terms of dynamic model. The tracking errors are defined, and the robust controller is designed for both the mobile platform and the manipulator to guarantee that the tracking errors go to zero asymptotically without the knowledge about system parameters and the external disturbance. The mobile robot and the manipulator can track the curved line at the bounded desired velocity. The simulation results show that the controller is possible to implement in the practical field. However, the experimental results have not yet to be done at this time of this writing. They will be considered in the future work.

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