KHẢO SÁT MỐI QUAN HỆ GIỮA ĐỘ THẤM VÀ ỨNG SUẤT VỈA BẰNG MÔ HÌNH KẾT HỢP CƠ ĐỊA VÀ DÒNG LƯU CHẤT

INVESTIGATING THE RELATIONSHIP BETWEEN PERMEABILITY AND RESERVOIR STRESS USING A COUPLED GEOMECHANICS AND FLUID FLOW MODEL

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TÓM TẮT

Trong hai thập niên vừa qua, các phương pháp số liên quan đến sự tương tác giữa biến dạng đá và hành vi dòng lưu chất đa pha đã được quan tâm nghiên cứu. Sự tương tác giữa cơ địa và việc khai thác lưu chất có thể ảnh hưởng đáng kể đến trạng thái ứng suất và dòng chất lỏng trong vỉa. Nhiều nhà nghiên cứu đã thử xây dựng một mô hình kết hợp hoàn chỉnh để phân tích vấn đề này. Tuy nhiên cho đến nay vẫn còn một trở ngại trong việc tích hợp biến độ thấm vào trong mô hình tích hợp, chủ yếu là do chưa nêu lên được mối quan hệ giữa độ thấm và ứng suất vỉa. Bài báo này sẽ trình bày một mô hình kết hợp hoàn chỉnh và cho thấy tầm quan trọng của độ thấm nhạy cảm với ứng suất trong vỉa hướng tâm, đặc biệt là phụ thuộc vào giai đoạn sụp lún cuối cùng và kết quả cố kết.

ABSTRACT

In the last two decades, there has been strong emphasis on numerical formulations related to the interaction between rock deformation and multiphase fluid flow behaviour in hydrocarbon reservoirs. This coupled interaction between geo-mechanics and fluid production can significantly influence both the stress state and fluid flow in the reservoir. Many researchers have tried to build a fully coupled model for analysis of the problem. However until now there has been a gap in resolving the full integration of the permeability variable on a coupled model; generally because of the lack of a relationship between permeability and reservoir stress. This study will present a fully coupled model and show how important stress-sensitive permeability is on a radial reservoir, particularly in terms of the final subsidence and consolidation results.

1. INTRODUCTION

When hydrocarbon fluids or water are produced from regions in the Earth near the surface, fluid pressure provided by these fluids is often reduced. As a result of reduction in fluid pressure, there is an increase in reservoir effective stress. Subsequently, the permeability of a reservoir will change because of the altering geometric structure of the porous media. Unlike a conventional simulator which analyses only the fluid flow with constant permeability; a fully coupled model can be used to not only simulate the fluid flow but also the behaviour of the rock particularly with regard to subsidence and compaction of the reservoir.

In the last century, many researchers have tried to build a fully coupled model to describe this problem. Previously, most authors investigating the coupled model have not included the relationship between permeability and stress. It can be honourably listed as Terzaghi (1925) who is considered the first author who resolved the coupled behaviour of water and soil in terms of material consolidation. Biot (1941) focused on extending the Terzaghi's theory into 3 dimensions. Following that work, coupled models were extended into other disciplines eg petroleum engineering, civil engineering, geotechnical engineering and rock mechanics. This would include Sandhu and Wilson (1969) and Ghaboussi and Wilson (1973) who produced some of the earliest coupled hydromechanical models. In recent years, Gutierrez et al. (1994) presented the general equations and the theory behind a fully coupled analysis for hydrocarbon reservoir compaction and subsidence. They showed that compaction drive could not be properly represented by simply adjusting the value of rock compressibility used in a traditional reservoir simulation.

From experimental work, Mattax et al. (1975) has shown that the average permeability decreased by almost 40% when the applied hydrostatic compaction pressure was increased from 65 to 3500 psi. Other results from this research were interesting in that the permeability response would have a more directly measurable effect on reservoir performance than porosity when overburden pressure was applied to a sample. However, this research was conducted with unconsolidated and friable sand cores. Therefore, the reliability and accuracy, of these findings with respect to consolidated cores needs to be confirmed. In general from an experimental point of view, Al Harthy et al. (1998) showed that the petrophysical properties such as permeability, capilliarity, porosity, resistivity and relative permeability are influenced by the state of stress in the reservoir. This research is being further developed to enable a range of petrophysical and flow measurements under true triaxial stress and elevated pore pressure conditions.

In Vietnam, there is very little research relating to "coupled theory" particularly in ground subsidence under reservoir fluid production conditions. Trinh and Delwyn (1996) presented some results from subsidence measurements due to pumping of ground water in the City of Hanoi. They showed subsidence, in the central and south-eastern part of the city of Hanoi, was a serious problem with settlement about 20 to 35mm/year. These results reasonably match subsidence measurements made at specific well locations. However, this model was produced using uncoupled theory. This research should be extended adding more information about the soil coefficient of permeability varying in both the horizontal and vertical directions and including stress sensitive permeability.

Although, there are numerous authors working on fully coupled modelling, a large gap still remains in integrating the influence of permeability on the coupled effect because of the lack of a relationship for stress and permeability. Knowledge of stress-sensitive permeability within a reservoir simulation and the influence of this parameter on results will be helpful for reservoir engineers in better making management decisions. The emphasis of this work has been to study not only the mathematical equations for the fully coupled model in an oil simulator, but also to investigate the stress sensitive permeability using in the coupled geomechanics and fluid flow model

This paper mainly focuses on a fully coupled well model by presenting:

- An overview of coupling theories for radial flow in reservoir engineering.

- The sensitivity of permeability to stress in a fully coupled model.
- A simplified case study under standard reservoir conditions.

The equations are based on the platforms of continuum theories of multiphase material. Results of this work can be specifically applied to studying compaction and subsidence in a reservoir simulation.

2. NUMERICAL SOLUTION OF THE GOVERNING EQUATIONS

Constitutive laws reflect the internal constitution of a material, by defining specific types of material behaviour. They are fundamental in the sense that they are the starting point for studies in the discipline of elasticity, plasticity, and various idealized fluids. The mechanical constitutive laws of a material specify the dependence of stress in a body on kinematic variables such as the strain tensor or the rate of deformation or pressure with the effective stress. The system of equations obtained by applying the conservation laws are not fully determinate, i.e., the system has more unknowns than the number of equations. To close the system, we need additional equations relating stress to deformation. This will close the system of equations, i.e.; the number of unknowns will equal the number of equations. Boundary conditions are to be applied on the boundary which will be transmitted into the material by the rules determined by the system of partial differential equations. Boundary conditions can be given as stresses or solid displacements or fluid pressure prescribed at the boundary. As mentioned, the formulations of coupled theories between fluid and solid applied to radial flow in reservoir engineering are based on the mass balance and momentum balance principles of continuum theory. Unfortunately, the general analytical solution for this equation has not been resolved except for a simplified case. Despite this, a numerical method can be applied to achieve the general solution. The Galerkin finite element method is chosen because of its ability to handle anisotropic and heterogeneous regions with complex boundaries (Young and Hyochong, 1996).

In the Galerkin method, the unknown variable pressure and displacements can be approximated by a trial solution in space using of the shape function N and nodal values (P, u)

$$P = \sum_{i=1}^{n} N_i P_i \tag{1}$$

$$u_r = \sum_{i=1}^{n} N_i u_{ri} \tag{2}$$

$$u_z = \sum_{i=1}^{n} N_i u_{zi} \tag{3}$$

Therefore, in the coupled simulation, there are three principal degrees of freedoms at each node of the mesh.

Applying the time integration technique with Crank-Nicolson method described the coupled matrix system can be presented as following

$$\begin{bmatrix} 2M_2 & 2M_1 + \Delta t K_1 \\ K_2 & C_2 \end{bmatrix} \cdot \begin{bmatrix} u^{t+\Delta t} \\ P^{t+\Delta} \end{bmatrix} = \begin{bmatrix} \Delta t\left(F_1^{t+\Delta t} + F_1^{t}\right) + \left(2M_1 - \Delta t K_1\right)P^t + 2M_2 u^t \\ \left(F_2^{t+\Delta t} + F_2^{t}\right) - \left(K_2 u^t + C_2 P^t\right) \end{bmatrix} \tag{4}$$

Where

$$M_1 = \int \frac{1}{\alpha\beta}\frac{\mu\phi}{K} C_t N_i N_j dR$$

$$K_1 = \int \frac{\partial N_i}{\partial r}\frac{\partial N_j}{\partial r} dR + \int \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z} dR$$

$$M_2 = \int \frac{1}{\beta}\frac{\mu\phi}{K} N_i \left(\frac{\partial N_j}{\partial r} + \frac{\partial N_j}{\partial z}\right) dR$$

$$F_1 = 2\pi r N_i \frac{\partial P}{\partial r} n_r \bigg|_B + 2\pi r N_i \frac{\partial P}{\partial z} n_z \bigg|_B$$

$$K_2 = \int \frac{\partial N_i}{\partial g} . D . \frac{\partial N_j}{\partial g} dR$$

$$C_2 = (1-\phi_0)\int N_j \cdot \left(\frac{\partial N_i}{\partial r} + \frac{\partial N_i}{\partial z} \right) dR$$

in which

$$\dot{P} = \left\{ \dot{P}_1, \dot{P}_2, \dot{P}_3, \dot{P}_4 \right\}^T$$

$$u = \{u_1 \ v_1 \ u_2 \ v_2 \ u_3 \ v_3 \ u_4 \ v_4\}^T$$

$$\sigma' = \{\sigma_r', \sigma_\theta', \sigma_z', \tau_{rz}'\}^T$$

$$\varepsilon = \{\varepsilon_r, \varepsilon_\theta, \varepsilon_z, \tau_{rz}\}^T = \left\{ \frac{\partial u_r}{\partial r}, \frac{u_r}{r}, \frac{\partial u_z}{\partial z}, \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right\}^T$$

Note that, porosity and permeability will be updated at each time step on each element. Certainly, the matrix of eq.(4) is non-linear and unaxisymmetric. This matrix must be completely constrained by initial and boundary conditions, before a solution is obtained.

The detail in deriving this fully coupled equation can be found in Ta and Hunt (2005).

3. NUMERICAL EXAMPLES

3.1. Model description

This example simulates the subsidence and compaction of oil reservoirs within a radial reservoir model. The production zone in this model is shown in Figure 2, highlighted in green. The reservoir in this model is thick compared to the depth, perforated zone and a real field scale example. Hence, to simulate full reservoir depletion, only 200 days of oil pumping is required.

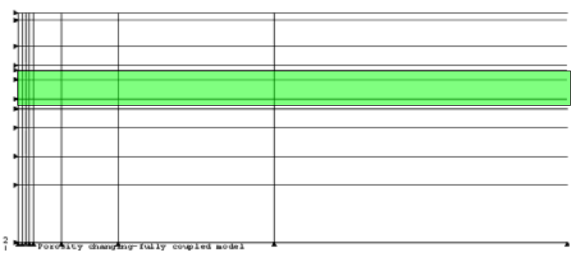

Fig.1: The axisymmetric well model used. Green highlight indicates the production zone.

The coupled model analysis is written within the Matlab programming environment and solves problems involving fluid flow through a saturated elastic porous medium under transient analysis. Experimental data will be used in the next stage of this research to compare with the numerical results for the example given.

The example considers an axisymmetric model of an oil well and the surrounding material, as shown in figure 1. In this model, a fully penetrating well of radius r_w is producing a single phase fluid at a constant rate q, from a saturated reservoir. The reservoir is assumed to be homogeneous and isotropic with a boundary being restrained from any radial displacement at the producing wellbore, but allowing free displacement in the vertically direction. A coarse mesh is selected for the illustrative purpose of this example. No mesh convergence study has been performed.

At t = 0, the reservoir pressure is equal to the initial reservoir pressure. Due to the application of an axisymmetric model, we used isoparametric elements for the porous media in all regions of the radial model which had 204 nodes meshed into 176 elements.

3.2. Material properties of reservoir

Material properties used are given in table 1.

Table 1: Material properties of reservoir in the simulation

Material properties	**Symbol**	**Values**	**Field unit**
Initial porosity	ϕ	0.15	-
Poison's ratio	ν	0.25	-
Initial permeably	K	300	mD
Young modulus	E	5.6 E6	psi
Fluid density	ρ_f	49.8	p/ft^3
Fluid compressibility	C_f	15.E-06	psi^{-1}
Solid compressibility	C_s	7.0E-06	psi^{-1}
Initial pressure	P_i	4800.0	psi
Production rate	q	1200	Stb/d
Well radius	r_w	0.5	ft
External boundary	R	7932	ft
Depth	z	4798	ft

3.3. Permeability data

The data for monitoring the change of effective stress and pore pressure are very important in coupled simulations as the models require an explicit relationship between permeability and the state of stress in the reservoir during computation. Until now, conventional laboratory methods are usually used for creating such relationships. Because each reservoir has specific properties, it is not possible to apply a general relationship for all coupled models. In addition, the chosen laboratory methods are based on a particular purpose and budget of those projects. For this study, the variation of permeability under axial stress, non-hydrostatic conditions, has been used. These results are obtained from the work on Red-Wildmore sandstone by Gang and Maurice (2003). This relationship is used for illustrative purposes.

3.4. Results and discussion

3.4.1 Subsidence investigating with different initial permeability models.

Figure 2 shows the subsidence at the centre of the model, the central sink of the well area after 200 days of production. The two models presented have the same production rate. The first model which has a lower initial permeability has a lesser degree of subsidence approximately 1.1ft. In contrast, the other model which was simulated with a higher initial permeability has, as expected, a larger degree of subsidence. After 200 days, the calculated subsidence from the second model is approximately 20% higher compared to the result from the first model. The subsidence is projected to be much higher with increased production time and production rate. Obviously, the permeability parameter presents is having significant effect on subsidence when under the influence of a stress sensitive reservoir. The effect of permeability compared to other parameters is analysed in the next section.

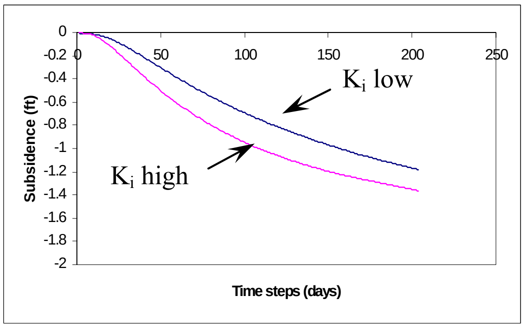

Fig. 2: Subsidence due to production at center sink. K_i = initial permeability.

3.4.2. Influence of stress on the permeability

From previous laboratory based experimental work it was been shown that the permeability variation has a more directly measurable effect on reservoir performance than porosity.

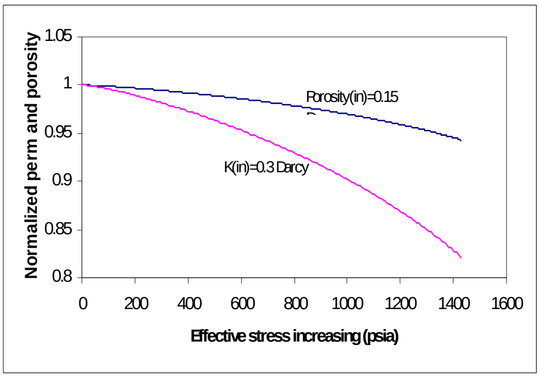

Fig. 3: Normalized permeability and porosity (current by initial) plotted as function of effective stress. The initial porosity and permeability values are given.

The results obtained in the modelling work are in agreement with this finding, as shown plotted in figure 3 where normalized porosity and permeability versus increasing effective stress are given . The results show a slight reduction in porosity when effective stress increases from 0 to nearly 1500 psia. On the other hand, normalized permeability is heavily influenced by effective stress. Therefore, not only does the porosity change but also permeability changes considerably following a long production time over the life of the reservoir. These findings should be taken into account when using the conventional simulator for history matching.

3.4.3. Pore pressure reduction

A simulation of reservoir depletion was also run for both uncoupled and coupled models applying the same initial conditions for fluid production. Also taking into account the rock deformation and its effect within the coupled model. The results in figure 4 shows the differential variation in pore pressure between the uncoupled pore pressure and the coupled pore pressure models results. The results suggest the pore pressure variation is the same initially. However, after 40 days, the reduction in pore pressure in the uncoupled model is significantly greater than the results from the coupled model. The comparative results suggest that the traditional well test procedure would be suitable at the early production time and acceptable for both cases because the fluid rate is not influenced. The effect of the coupled phenomena has on subsidence in the long term is presented in the next section.

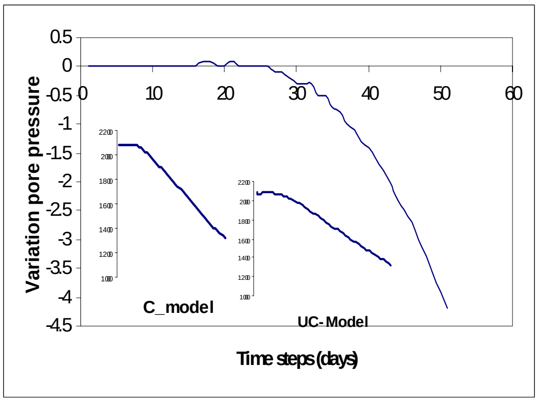

Fig. 4: Difference between the pore pressures for uncoupled minus the coupled models plotted against production time ($\Delta P = \Delta P_{uc} - \Delta P_c$)

3.4.4 Increase in effective stress

The result of fully coupling the model for reservoir simulation, shows that when fluid is withdrawn from the reservoir, the pore pressure will be reduced. In turn, effective stress will be increased. Subsequently, the reservoir will deform causing changes in pore structure which closely relate to permeability changes in the reservoir.

Figure 5 clearly presents the increase in effective stress in the reservoir after nearly 200 days production. At the beginning of production, there is no sign of an increase in effective stress. However, after 30 days, the effective stress increases dramatically reaching about 1400 psia after 200 days. This result should be considered for casing design and planning of reservoir development.

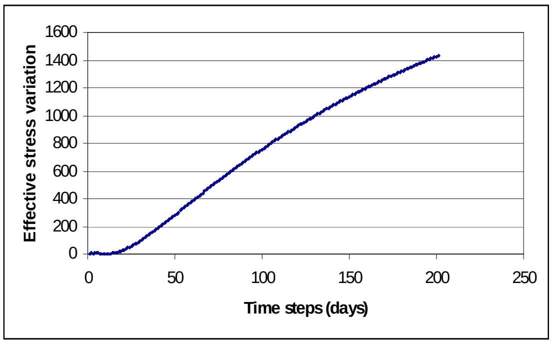

Fig. 5: Effective stress variation with increasing production time.

4. CONCLUSIONS

A numerical simulation using the Galerkin finite element method has been applied to model the subsidence effect in a simplified radial reservoir. The governing equations represent the behaviour of fluid flow and the deformation of the rock. These equations have been taken into account and the relationship between increasing effective stress and permeability, based on the experiment relations for specific rock types have been applied. The influence of changing permeability on effective stress is more important than the effect of reducing porosity. The results, of this stress sensitive permeability study show that the analysis is important for predicting subsidence and planning for reservoir development.

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REFERENCES

1. Terzaghi, K. Erdbaumechanik auf bodenphysikalischer grundlage, Franz Deutikle, Leipzig und Wien (1925).
2. Terzaghi, K.: Theoretical Soil Mechanics, Wiley, New York (1943).
3. Biot, M.A. General theory of three-dimensional consolidation, Journal of Applied Physics 12 (1941).
4. Sandhu R S and Wilson E L. Finite-element analysis of seepage in elastic media. J Engineering Mech Div ASCE 95 (1969), pp. 641-652.
5. Ghaboussi J and Wison E L. Flow of compressible fluids in porous elastic media. Int J Numer Methods Eng 5 (1973), pp. 419-442.
6. Mattax C.C and McKinley R.M and Clothier A.T,. Core analysis of unconsolidatied and friable sands. Journal of Petroleum technology (1975), pp. 1423-1432.
7. Gutierrez, M. Fully coupled analysis of reservoir compaction and subsidence, paper SPE, 28900, presented at the SPE European Petroleum Conference, London, 25-27 October (1994).
8. Young W.K and Hyochong B,. The Finite Element Method using Matlab, CRC Press, The United State of America (1996)
9. Trinh M.T. Delwyn G.F. Subsidence in the city of Hanoi, Viet Nam. Proceeding of the Canadian society of Civil Engineering Annual Conference, Regina, Saskatchewan, Canada. June 2-5 (1999).
10. McKinley J. Coupled consolidation of solid, infinite cylinder using a Terzaghi formulation, Computer and Geotechnics 23 (1998), pp. 193–204.
11. Al-Harthy, S.S. Dennis, J. Jing X.D. and Marsden, J.R. Petrophysical properties under true-triaxial stress for hydrocarbon recovery prediction, paper SPE, 39770, presented at the 1998 SPE Permian Basin Oil and Gas recovery conference held in Midland, Texas, 25-27 March (1998).
12. Gang, H. and Maurice, B. D. Description of fluid flow around a wellbore with stress dependent porosity and permeability. Journal of Petroleum science and engineering (2003).
13. Sansour, C. Fundamental of Non-linear Computational Mechanics, Australia, Lecture Notes, The University of Adelaide, Australian School of Petroleum (2004).
14. Behrenbruch, P. Biniwale, S. Characterisation of classic depositional environments and rock pore structure using the Carman_Kozeny equation: Australian sedimentary basins. Journal of Petroleum science and engineering (2005).
15. Ta, Q.D. and Hunt, S.P. Sansour, C. Applying fully coupled geomechanics and fluid flow model theory to petroleum wells. Alaska Rocks 2005, The 40th U.S. Symposium on Rock Mechanics (USRMS), held in Anchorage, Alaska, June 25-29 (2005).
16. Ta, Q.D. and Hunt, S.P. Investigating the relationship between permeability and porosity using in the coupled geomechanics and fluid flow model (accepted). The 3rd International Workshop on Hanoi Geo-engineering 2005 - Integrated GeoEngineering for a Sustainable Infrastructure Development, held in Ha Noi, Viet Nam (2005).