CALCULATION OF SELF AND MUTUAL INDUCTANCES OF INDUCTION FURNACES AND MOLTEN METAL FEEDERS USING LOW FE MODELS

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ABSTRACT

The paper deals with a possible way of computing inductances for circuit models of low-frequency complex devices from the magnetic field distribution. Computations are carried out by the FEM-based professional program FEMLAB supplemented with several single-purpose user procedures in Matlab written by the authors. The principle and results are illustrated on an example of a zinc feeder.

1. INTRODUCTION

Modeling of induction furnaces and other induction equipments such as induction feeders and molten metal pumps belongs to commonly solved task. Both in industry and research this task is often fixed by neglecting displacement currents and higher harmonics and solved by finite elements method. The model starting from the modified Maxwell equations is called the low frequency harmonic FE model. The solution obtained from this model involves local values of electromagnetic field quantities. Other derived local and integral quantities (such as heat power density and total heat power) may also be easily obtained from the solution. However, electrical lumped parameters, such as resistances or self- and mutual inductances of particular coils may also be required. These parameters are used, for example, for design of the source of power. Unfortunately, getting these parameters from solution of electromagnetic field is not trivial, particularly in the case of several coils and nonlinearities.

The paper deals with calculation of self- and mutual inductances from the model of electromagnetic field solved by the finite element method. Its main ideas are illustrated on an example of a zinc feeder.

2. INTRODUCTION TO ZINC FEEDER

Zinc feeder is a special induction device for pumping and dosing of molten zinc. Its schematic arrangement is depicted in Fig. 1. Liquid metal flows from the container or furnace through the inlet into a ring-shaped tank. Several coaxial inductors wound round the magnetic core and carrying harmonic currents generate periodic magnetic field producing in molten metal eddy currents flowing in the circumferential direction. These currents (in interaction with the primary magnetic field) produce the Lorentz forces making molten metal rise upwards, to the level of the outlet. Liquid metal then flows through the outlet out of the feeder into a mould.

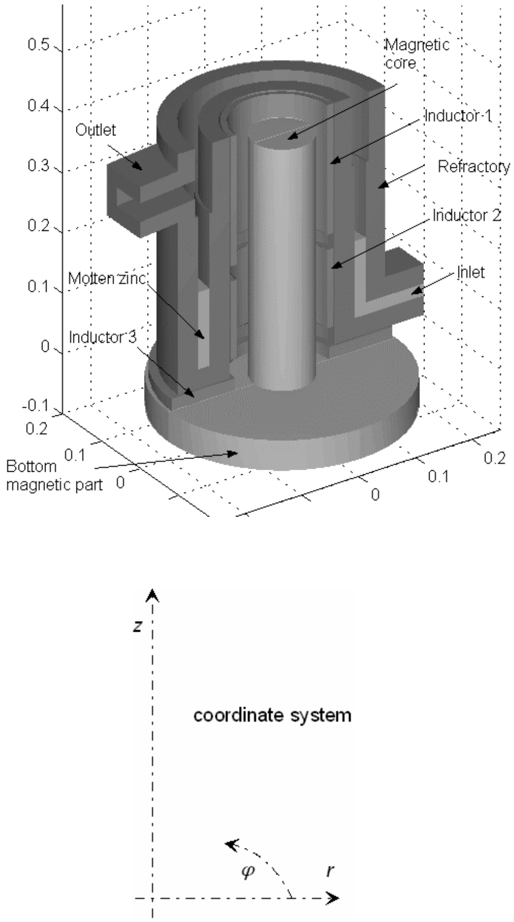

Fig. 1: Basic arrangement of the feeder

The feeder works in two main regimes: regime of dozing and regime of heating. During the regime of dozing magnetic field in the system is produced by inductors **2** and **3** (the Lorentz forces acting mainly in the bottom layers of melt are high). During the regime of heating the molten metal is kept at an approximately constant temperature (its cooling and eventual solidification might damage the refractory) by means of induction heating. Magnetic field in this regime is produced by inductors **1** and **2**. Electromagnetic forces are now low and do not push the metal out of the feeder. In order to find the time of filling the mould it is necessary to know dynamics of the device (at least at the stage of design) as accurately as possible.

3. MATHEMATICAL MODEL

a. Electromagnetic field model

Induction devices are usually supplied by currents of frequency low enough, that the displacement currents can be omitted. Electromagnetic field can be than described by equation

$$\mathrm{curl}\left(\frac{1}{\mu}\mathrm{curl}\,\boldsymbol{A}\right)+\gamma\cdot\frac{\partial\boldsymbol{A}}{\partial t}=\boldsymbol{J}_{\mathrm{ext}} \qquad (1)$$

where $\boldsymbol{A}$ defined the vector potential defined as: $\mathrm{curl}\,\boldsymbol{A}=\boldsymbol{B}$.

As far as the magnetic core is not expected to be fully saturated, all material parameters may be considered approximately constant. The model of the feeder is then linear. This simplifying assumption allows the computations of electromagnetic field to be carried out in a complex plane using the harmonic analysis.

Distribution of the electromagnetic field is now described by equation

$$\mathrm{curl}\left(\mathrm{curl}\,\underline{\boldsymbol{A}}\right)+\mathrm{j}\,\mu\omega\gamma\,\underline{\boldsymbol{A}}=\mu\underline{\boldsymbol{J}}_{\mathrm{ext}} \qquad (2)$$

The harmonic analysis enormously reduces the computation time, effort and stability. However, when a magnetic core is present, it is often not fully possible to neglect its saturation. In such a case we use a special model of nonlinear harmonic analysis. This model also considers a constant permeability, but its local value depends on the local magnetic flux density amplitude. The electromagnetic field is then described by equation.

$$\mathrm{curl}\left(\frac{1}{\mu\left(\left|\underline{\boldsymbol{B}}\right|\right)}\mathrm{curl}\,\underline{\boldsymbol{A}}\right)+\mathrm{j}\,\omega\gamma\,\underline{\boldsymbol{A}}=\underline{\boldsymbol{J}}_{\mathrm{ext}} \qquad (3)$$

The above models of electromagnetic field can be solved by the finite element method. In the next text we will discuss the ways of determination of inductances starting from them. But because of rather limited space, only computation starting from the last model will be illustrated on one example. Nevertheless, this example involves most of the phenomena that can complicate the calculation: several current carrying inductors, ferromagnetic core and a zinc area with surface currents.

b. Computation of inductances

Inductance of a solitary coil in linear surroundings can easily be computed from the total energy of electromagnetic field and its value follows from formula

$$L = \frac{2W}{I^2} \tag{4}$$

In case that the task is nonlinear, the inductance must satisfy equations

$$\frac{\mathrm{d}W}{\mathrm{d}t} = UI \tag{5}$$

$$U = \frac{\mathrm{d}LI}{\mathrm{d}t}. \tag{6}$$

valid in circuit models with lumped parameters. Combining both above formulas, the differential of energy W can be expressed as

$$\mathrm{d}W = I\,\mathrm{d}LI \tag{7}$$

Consider now a linearized problem obtained by setting the local permeability to its instantaneous constant value. Inductance in such a modified problem is then independent of current. The differential of energy is given by

$$\mathrm{d}W = LI\,\mathrm{d}I \tag{8}$$

The second differential of the energy is

$$\mathrm{d}^2 W = L\left(\mathrm{d}I\right)^2 \tag{9}$$

The self-inductance can be then defined as a second derivative of magnetic field energy of a linearized system due with respect to current

$$L = \frac{\partial^2 W}{\partial I^2} \tag{10}$$

This derivation can easily be extended to definition of self- and mutual inductances in systems with more inductors. That is why this definition can be useful even in linear problems. The derivation for the case of two coils as follows

In circuit models with two magnetically coupled coils the following equations have to be satisfied

$$\frac{\mathrm{d}W}{\mathrm{d}t} = U_1 I_1 + U_2 I_2 \tag{11}$$

$$U_1 = L_{11}\frac{\mathrm{d}I_1}{\mathrm{d}t} + M_{12}\frac{\mathrm{d}I_2}{\mathrm{d}t} \tag{12}$$

$$U_2 = L_{22}\frac{\mathrm{d}I_2}{\mathrm{d}t} + M_{12}\frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{13}$$

(all coefficients expressing the self- and mutual inductances being assumed constants). From these equations a differential of magnetic field energy can be expressed as

$$\mathrm{d}W = L_1 I_1\,\mathrm{d}I_1 + M_{12}\left(I_1\,\mathrm{d}I_2 + I_2\,\mathrm{d}I_1\right) + L_2 I_2\,\mathrm{d}I_2 \tag{14}$$

Now the second differential reads

$$\mathrm{d}^2 W = L_1\,\mathrm{d}I_1^2 + 2M_{12}\,\mathrm{d}I_1\,\mathrm{d}I_2 + L_2\,\mathrm{d}I_2^2 \tag{15}$$

and the inductances can be expressed as

$$L_{11} = \frac{\partial^2 W}{\partial I_1^2},\; M_{12} = \frac{\partial^2 W}{\partial I_1 \partial I_2},\; L_{22} = \frac{\partial^2 W}{\partial I_2^2} \tag{16}$$

where the derivatives are carried out on a linearized model with locally time invariable permeability.

This model and definition is applicable in both linear models and models with variable permeability, but not in models with hysteresis. When hysteresis occurs, magnetic field cannot be modeled by only self- and mutual inductances in circuit models. A more complicated model involving some resistances must then be implemented because of losses in a ferromagnetic material.

4. CIRCUIT MODEL OF ZINC FEEDER

The zinc feeder was modeled in the regime of dosing. Self- and mutual inductances of two operating inductors and a short-circuited winding represented by the ring shaped zinc volume were computed. The computation was carried out for various zinc levels in the feeder.

First we calculated the distribution of harmonic electromagnetic field. The computation was nonlinear because the permeability depends on the resultant field distribution. The field distribution provided the distribution of permeability in the magnetic core and current density in zinc. These results were used for further computations of electromagnetic field for several different values of total current in particular coils (and, consequently, in zinc) that were carried out in order to obtain the total magnetic energy for different currents. Its second derivatives with respect to the corresponding currents then provide the above self- and mutual inductances. We used the following formulas:

$$\frac{\partial^2 w_E}{\partial I_i^2} = L_i \doteq \frac{w_E(I_i+\Delta I) - 2w_E(I_i) + w_E(I_i - \Delta I)}{\Delta I^2} \tag{17}$$

$$\frac{\partial^2 w_E}{\partial I_i \partial I_j} = M_{ij} = \frac{1}{4\Delta I^2}\left[\begin{bmatrix} w_E(I_i+\Delta I, I_j+\Delta I) \\ -w_E(I_i+\Delta I, I_j-\Delta I)\end{bmatrix} - \begin{bmatrix} w_E(I_i-\Delta I, I_j+\Delta I) \\ -w_E(I_i-\Delta I, I_j-\Delta I)\end{bmatrix}\right] \tag{18}$$

where I is the basic current and ΔI is the current difference. Due to linearity of the model the basic current can be chosen quite arbitrarily.

To minimize the number of magnetic field energy computations and also the numerical error, the point of derivation was chosen $I_i = 0$ for self- and $\{I_i, I_j\} = \{\Delta I, -\Delta I\}$ for mutual inductances and the current difference ΔI for mutual inductances was half of the difference for the self-inductance. With the fact that $w_E(I) = w_E(-I)$, the formulae for computation of inductances can then be simplified to the form

$$\frac{\partial^2 w_E}{\partial I_i^2} = L_i = \frac{2\, w_E(\Delta I)}{\Delta I^2} \tag{19}$$

$$\frac{\partial^2 w_E}{\partial I_i \partial I_j} = M_{ij} = \frac{w_E(\Delta I, 0) + w_E(0, \Delta I) - w_E(\Delta I, -\Delta I)}{\Delta I^2} \tag{20}$$

The electromagnetic field was computed as two dimensional because its geometry is almost completely axi-symmetric. Computation was done in the complex domain (using the harmonic analysis) by FEM-based professional code FEMLAB supplemented with several single purpose codes written in Matlab by the authors.

The magnetic core was considered nonconductive. Its permeability is expected to be constant in time, but dependent on the effective value of local magnetic flux density. The dependence between relative permeability and effective value of magnetic field density is depicted in Fig.2

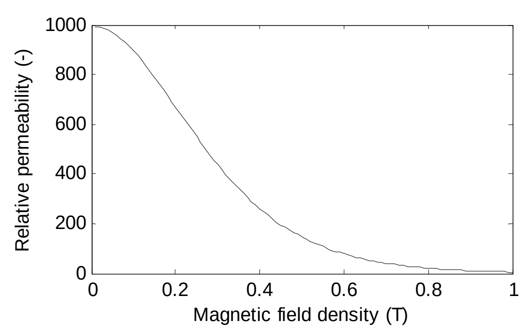

Fig. 2: Dependence of the relative permeability on magnetic flux density used in computations

The feeder was modeled in the operation regime of dosing. Therefore, only the two lower inductors are considered in the model. The inductors carry harmonic source currents. The influence of skin-effect in the inductors is neglected.

A resultant distribution of magnetic field is depicted in Fig. 3, for the medium level of zinc.

A circuit model of the feeder consists of three mutually magnetically coupled inductances and several resistors (Fig. 4). Each inductance represents either one of the inductors or the zinc volume:

inductance L_2 - inductor No. 2,
inductance L_3 - inductor No. 3,
inductance L_z - zinc.

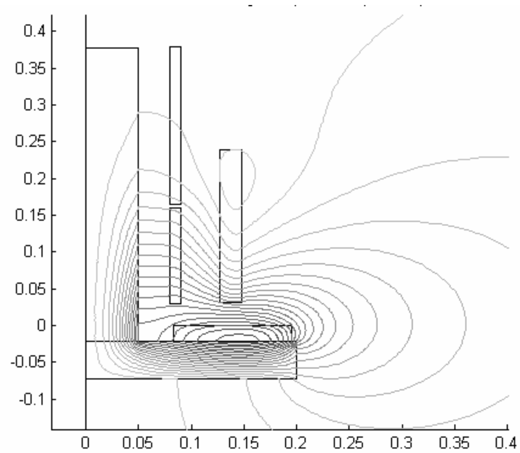

Fig. 3: Distribution of magnetic field for the medium level of zinc

Resistors respect heat losses in the inductors or zinc.

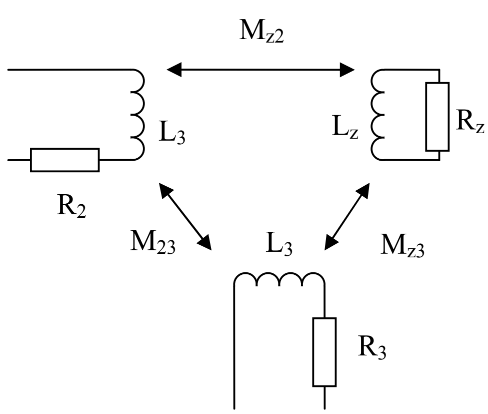

Fig. 4: A circuit model of the zinc feeder

During computation of the inductances, magnetic fields produced by several different currents have to be determined. The distribution of currents and permeability should be, however, the same for all these partial computations. Therefore, the real permeability and current density in zinc had to be found out. The permeability is than fixed and wanted current through zinc volume is obtained by multiplying the real current density by a suitable constant.

Inductances M_{2z}, L_z and M_{z3} are dependent on the zinc level. The zinc level in the feeder varies from 0.16 m to 0.33 m. These dependencies were also computed and are depicted in Fig. 5.

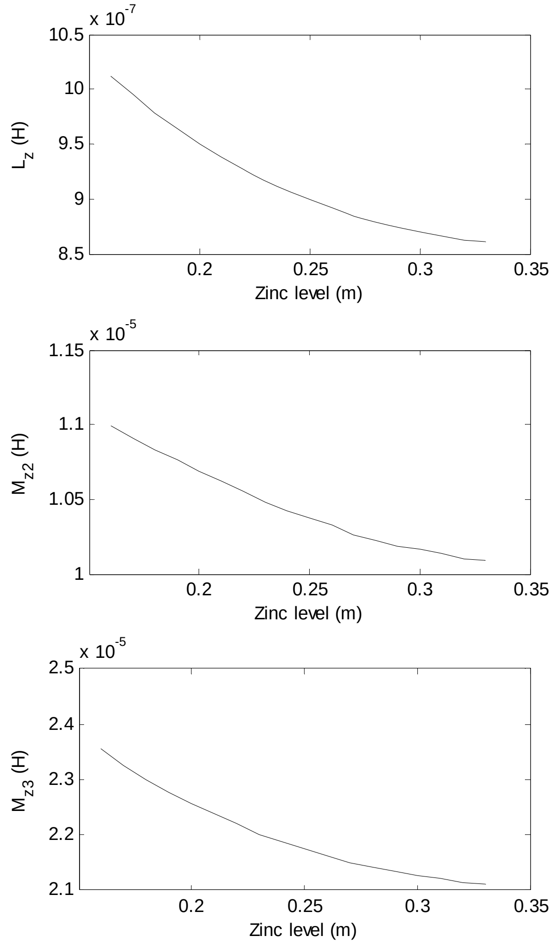

Fig. 5: Dependencies of inductance of zinc L_z and mutual inductances between zinc and inductors No. 2 and 3 (M_{z2} and M_{z3}) on the level of zinc in the feeder

The resultant values of inductances for zinc level 0.24 m were introduced into the circuit model of the feeder (see Fig. 6). Also resistances were computed. The resistance of zinc volume was computed from the heat power induced in the volume and resistances of coil were obtained from geometry and material resistivity, where the skin-effect in inductors was neglected again.

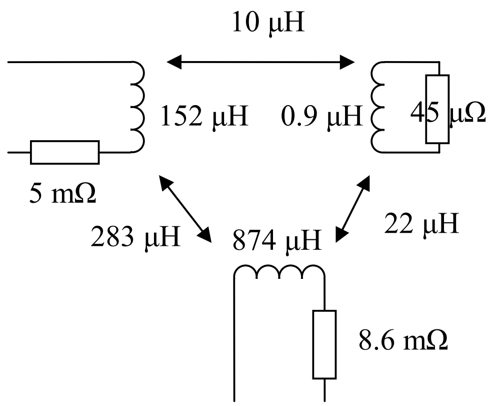

Fig. 6: The circuit model of the feeder computed for the zinc level of 0.24 m

5. CONCLUSION

One computation way of self and mutual inductances from magnetic field distribution is presented. The computation is feasible for both linear and nonlinear tasks. The method used for creation of the circuit model of a zinc feeder. This circuit model can be used for design of supply for the device.

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